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ON SEISMIC COMPUTATIONS, WITH APPLICATIONS, II[1]

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ABSTRACT

This paper is a continuation of a previous one given, with a somewhat similar title, in which are discussed the travel-times and other similar properties of the wave paths in a medium in which the seismic velocity is an integrable function of the depth. In this paper we consider similar properties of the waves from a point on the surface to a point in a "well" in such a medium.

In this paper, it proposed merely to continue the results of a previous paper[3] on the subject of wave paths and time-distance curves for media in which the seismic velocity is an integrable function of depth. The "time-depth curves"; i.e., the curves indicating the travel-times of seismic waves from a point of the surface to the points in a "well" are obtained in terms of "dimensionless" units in the so-called "linear" and "exponential" cases. It is hoped that these two cases, explained in some detail in this paper and in that cited above, will serve as a sufficient outline for other cases that may be proposed.

THE LINEAR CASE: $v=v_0+ah$.

In this case, the wave-paths are the circles of the family whose equation is:

$$(1) \qquad \left\{x - \left[(1 - p^2v_0^2)^{1/2}/ap\right]\right\}^2 + \left\{h + (v_0/a)\right\}^2 = 1/a^2p^2,^4$$

in which p is the parameter, the significance of which is that $\sin^{-1}(pv_0)$ is the angle of emergence of the wave path at the surface. In fact,

[1] Published by permission of the Board of Directors, Humble Oil and Refining Company.

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[3] Cf., On Seismic Computations, with Applications, I, GEOPHYSICS, pp. 9–22, Vol. I, No. 1, (1936).

[4] Loc. cit., equ. (12).

if, at any point along a wave-path we designate by θ the angle between the positive h-axis and the tangent direction of the path in the direction of propagation, then $\theta = sin^{-1}(pv)$, in which v is the velocity at the point. The coordinate system used is that shown in Fig. 1, where the "well" is represented by the line $x = d$.

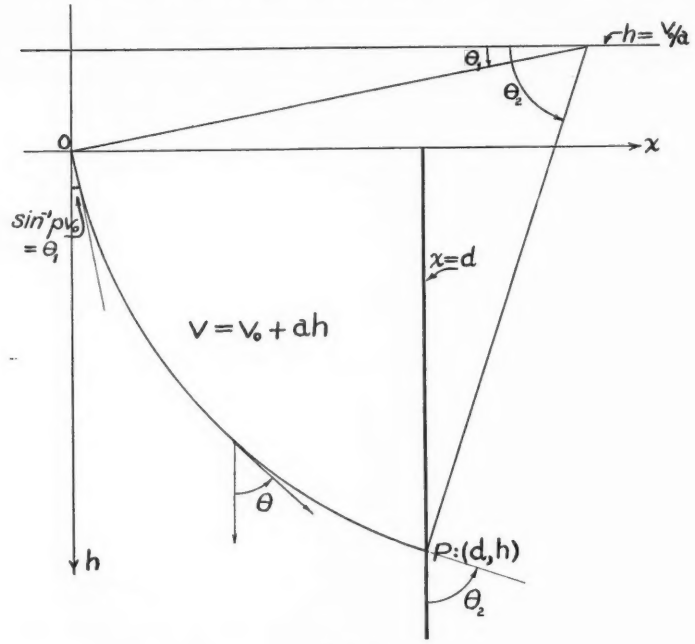

FIG. 1

Perhaps the best method of obtaining the desired results in this case is to introduce a polar coordinate system for each wave path in the following manner:[5]

$$\begin{aligned} \cos\theta/ap &= [(1 - p^2v_0^2)^{1/2}/ap] - x, \\ \sin\theta/ap &= h + (v_0/a). \end{aligned} \tag{2}$$

The travel time to the point $P:(d,h)$ from 0 is given by

$$t = \int_{\theta=\theta_1}^{\theta=\theta_2} ds/v, \tag{3}$$

[5] Here, as also in the next case, the analytic methods of the previous paper may be used directly. However, we felt that it is of some advantage to introduce the parameter θ, both for purposes of making the integrations simple and to indicate other possible points of view.

in which

$$ds = (1/ap)d\theta \tag{4}$$

and

$$v = v_0 + ah = \sin\theta/p. \tag{5}$$

Accordingly, equation (3) becomes

$$\begin{aligned} t &= (1/a)\int_{\theta_1}^{\theta_2} \csc\theta d\theta \\ &= (1/a)\{\log[\sin\theta_2/(1+\cos\theta_2)] - \log[\sin\theta_1/(1+\cos\theta_1)]\}. \end{aligned} \tag{6}$$

Obviously,

$$\sin\theta_1 = pv_0, \qquad \sin\theta_2 = p(v_0 + ah), \tag{7}$$

in which h is the ordinate of the point P. However, we also have

$$\begin{aligned} \cos\theta_1 &= (1 - p^2v_0^2)^{1/2}, \\ \cos\theta_2 &= \mp[1 - p^2(v_0 + ah)^2]^{1/2}, \end{aligned} \tag{8}$$

where the negative sign is to be used for $\theta_2 > \pi/2$ and the positive sign for $\theta_2 < \pi/2$;[6] i.e., when $h < [d^2 + (v_o/a)^2]^{1/2} - (v_o/a)$, the negative sign is to be used, and, when h is greater than this quantity, the positive sign holds. The case when h is equal to this quantity corresponds to $\cos\theta_2 = 0$. Thus, equation (6) can now be written in the form

$$\begin{aligned} t &= (1/a)\{\cosh^{-1}(1/pv_0) + \cosh^{-1}[1/p(v_0 + ah)]\} \\ &= (1/a)\cosh^{-1}\{1 + (1 - p^2v_0^2)^{1/2}[1 - p^2(v_0 + ah)^2]^{1/2}\} \\ &\qquad /\{p^2v_0(v_0 + ah)\}. \end{aligned} \tag{9}$$

From equation (1), we find that the value of p corresponding to the path through $P:(d, h)$ is:

$$p = 2d/\{a^2(d^2 + h^2)^2 + 4v_0(v_0 + ah)(d^2 + h^2)\}^{1/2}, \tag{10}$$

and, consequently,

$$\begin{aligned} [1 - p^2(v_0 + ah)^2]^{1/2} &= \mp[ah^2 + 2hv_0 - ad^2] \\ &\qquad /\{a^2(d^2 + h^2)^2 + 4v_0(v_0 + ah)(d^2 + h^2)\}^{1/2}. \end{aligned} \tag{11}$$

Thus, finally, equation (9) becomes:

$$t = (1/a)\cosh^{-1}\{1 + [a^2(d^2 + h^2)/2v_0(v_0 + ah)]\}. \tag{12}$$

[6] In any case, $0 < \theta_2 < \pi$.

Here we introduce the "dimensionless" quantities

$$\tau = at, \qquad \sigma = ad/v_0, \qquad \eta = ah/v_0, \tag{13}$$

and write (12) in the form

$$\cosh \tau = 1 + (\sigma^2 + \eta^2)/2(1 + \eta). \tag{14}$$

The appearance of this family of curves is shown in Fig. 2, in which we have also inserted (in dashes) the locus of "minimum-time" points, whose equation is $\cosh\tau = 1+\eta$.

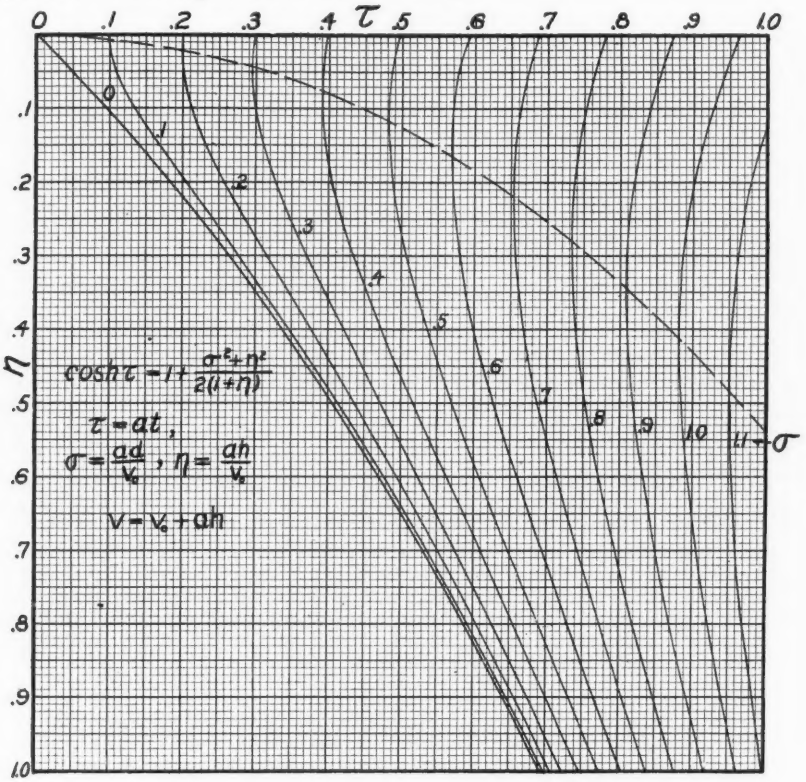

FIG. 2

THE EXPONENTIAL CASE: $v = v_0 \exp(ah)$.[7]

In this case, the family of wave paths has as its equation

$$\sin^{-1}\left[pv_0 \exp\,(ah)\right] - ax = \sin^{-1}\,(pv_0). \tag{15}$$[8]

[7] The symbol $\exp(x)$ is used to indicate e^x.

[8] Loc. cit., equ. (26).

It is to be noticed that, in the equation, the angle $\theta = \sin^{-1}[pv_0 \exp(ah)]$ is measured in the same way as explained in the linear case. The value of the parameter p for the curve of the family passing through $P:(d,h)$, (cf. Fig. 3), is

$$p = \sin ad/v_0[1 + \exp(2ah) - 2\exp(ah)\cos ad]^{1/2}. \tag{16}$$

Accordingly, we use equation (3), in which, since

$$\sin\theta = pv_0 \exp(ah), \tag{17}$$

$$ds = (1/a)\csc\theta d\theta, \tag{18}$$

and

$$v = (1/p)\sin\theta, \tag{19}$$

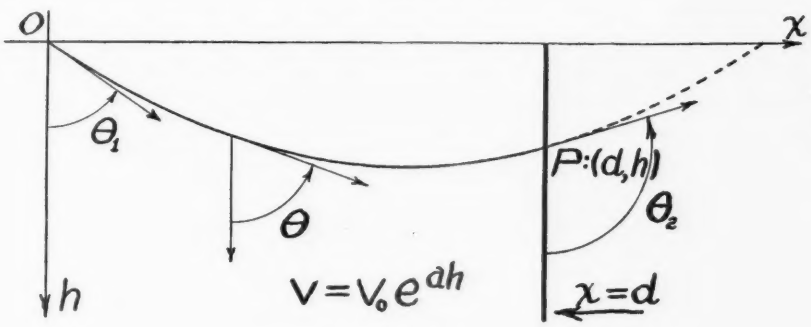

FIG. 3

we have

$$\begin{aligned} t &= (p/a)\int_{\theta_1}^{\theta_2} \csc^2\theta d\theta \\ &= (p/a)(\cot\theta_1 - \cos\theta_2). \end{aligned} \tag{20}$$

Here, of course,

$$\begin{aligned} \cos\theta_1 &= (1 - p^2v_0^2)^{1/2}/pv_0 \\ \cot\theta_2 &= \mp\,[1 - p^2v_0^2\exp(2ah)]^{1/2}/pv_0\exp(ah). \end{aligned} \tag{21}$$

From (16), we have

$$\begin{aligned} [1 - p^2v_0^2\exp(2ah)]^{1/2} = \mp\,&[\exp(ah)\cos ad - 1] \\ &/[1 + \exp(2ah) - 2\exp(ah)\cos ad]^{1/2}. \end{aligned} \tag{22}$$

Consequently, (20) becomes

$$(23)\quad t = (1/av_0)\{\exp(-ah)[1 + \exp(2ah) - 2\exp(ah)\cos ad]^{1/2}\}.$$

The "dimensionless" quantities that may be introduced here are:

$$(24)\qquad \tau = av_0t, \qquad \sigma = ad, \qquad \eta = \exp(ah);$$

and, as a result, equation (23) becomes:

$$(25)\qquad \tau = (1/\eta)(1 + \eta^2 - 2\eta\cos\sigma)^{1/2}.$$

These curves are shown in Fig. 4. It is to be noted that all the curves

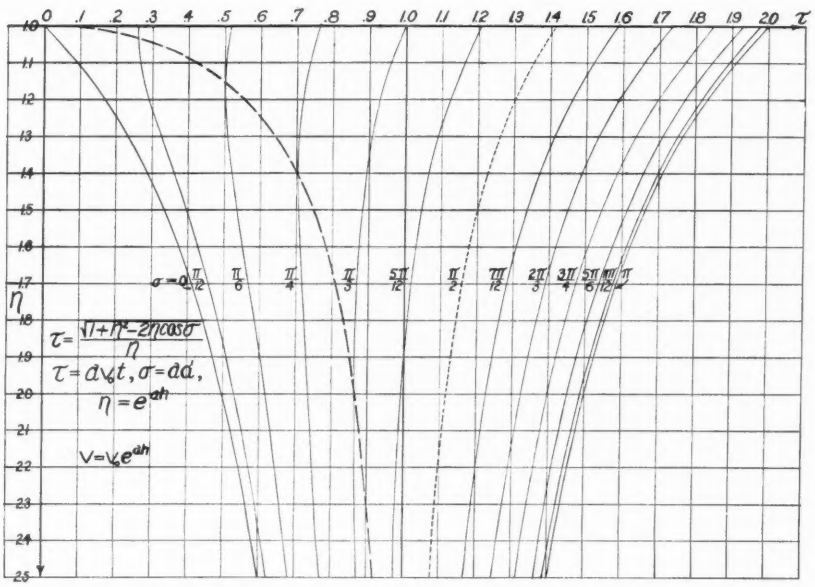

FIG. 4

have the line $\tau = 1$ as a common asymptote; that each of the curves, for which $\sigma < \pi/2$, has a minimum value for τ, and that the locus of these minima is the curve $\tau = (1/\eta)\ (\eta^2 - 1)^{1/2}$; that the curves for which $\sigma > \pi/2$ have no minima but are rather such that τ decreases monotonically with increasing η.

As a final point, we wish to remark that, considering t as a function

of h, in well-shooting from a fixed point on the surface, the value of the derivative of t with respect to h is given by the relation

$$dt/dh = \cos \theta_2/v. \tag{26}$$

In this result, θ_2 is the angle between the positive h-axis and the direction of propogation of the ray at the point in the well at depth h, and v is the seismic velocity there.

THE GROWTH OF COMPANY OWNED OPERATIONS IN GULF COAST GEOPHYSICAL PROSPECTING SINCE JULY 1930

E. E. ROSAIRE[1] AND K. RANSONE[1]

ABSTRACT

Graphical data are presented for the refraction seismograph, torsion balance, and reflection seismograph in the Gulf Coast over the period July 1930 to February 1936. For comparison, the variations in the price of 30° Baumé crude oil at Houston are presented for the same period.

The results indicate that: (1) The refraction seismograph was discarded before the 1931 minimum in the price of oil. (2) During this 1931 minimum, exploration of any kind was practically non-existent. (3) The 1933 minimum, on the contrary, seems to have effected contract operations more than company owned operations. (4) Company owned torsion balance operations seem to have reached a saturation point while contract torsion balance operations are rapidly decreasing from their 1934 peak. (5) Company owned reflection operations have shown a steady increase of ten crews per year while contract operations have shown a very irregular increase, averaging six crews per year, since 1932. (6) New contract reflection companies have appeared since 1930 at the rate of three per year. (7) The peak of contract reflection operations will probably be reached in 1937 or 1938, after which these operations will disappear in the same fashion as did the contract torsion balance operations.

INTRODUCTION

Most of us are aware of the fact that company owned geophysical explorations have been increasing, and it is generally assumed that at some future time the geophysical contracting companies will disappear from the field. The available data have been analyzed here with a view to investigating this trend for one particular petroleum producing province, where the activity is greater than in any other part of the U.S.A., and where petroleum production is dominated by the operations of large oil companies like the Humble, Shell, Gulf, Texas, Magnolia and Stanolind.

The data have been compiled from available scout reports. Since the making of these reports is a minor specialty of the local scout organization, the data are subject to frequent lapses, occasional glaring errors, and so are often erratic. As a result the charts show irregularities, generally due to the failure to report crews at all, or to reporting a given crew operating at several places in the same week. Careful checking by the junior writer has eliminated many irregularities which appeared in the preliminary analysis, but the results are still far from being precise. This is to be expected, however, since the data are gathered from several sources, few of which are exact.

[1] Independent Exploration Company, Houston, Texas.

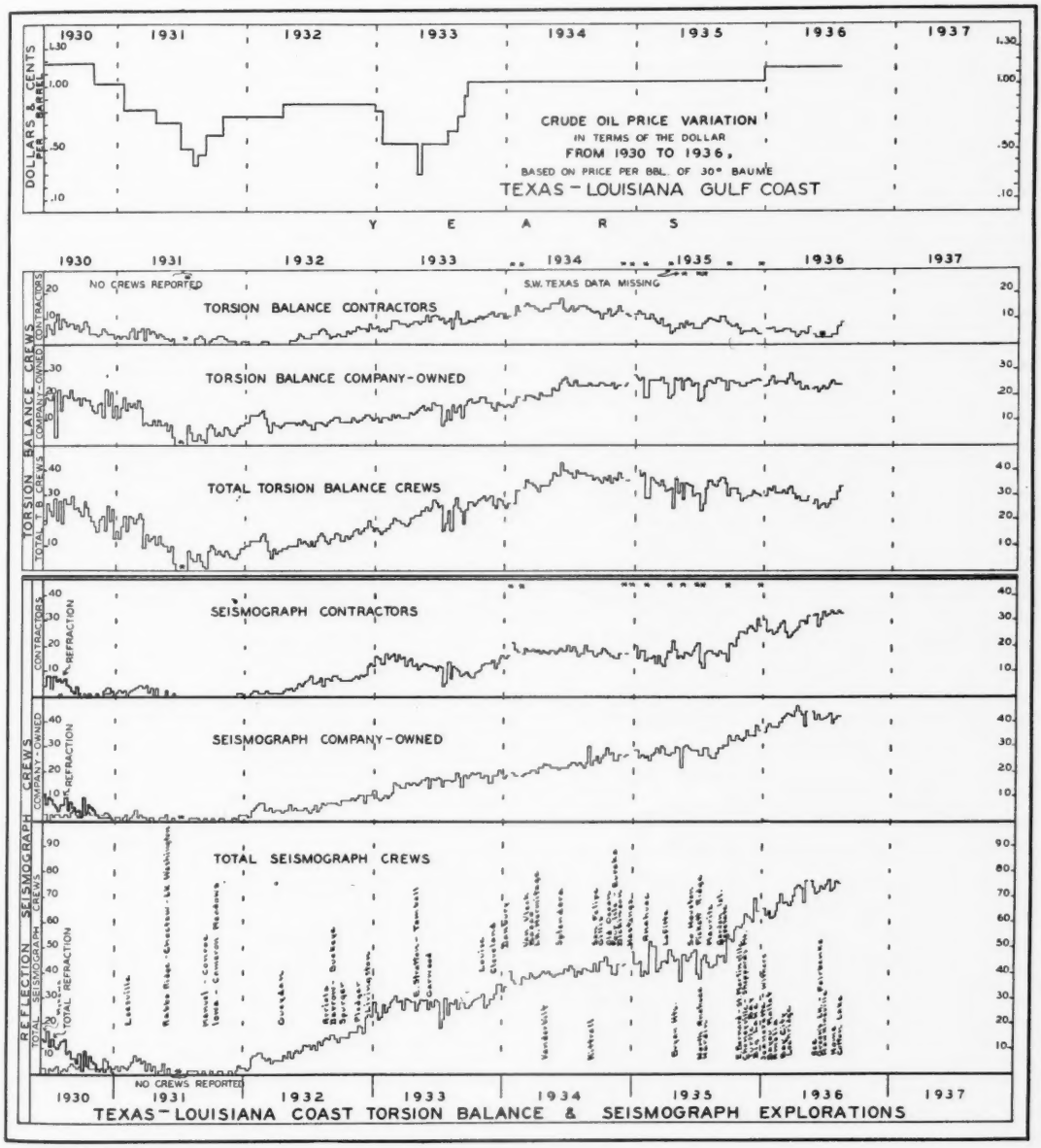
1930
1931
1932
1933
1934
1935
1936
1937
DOLLARS & CENTS PER BARREL
1.30
1.00
.50
.10
CRUDE OIL PRICE VARIATION
IN TERMS OF THE DOLLAR
FROM 1930 TO 1936,
BASED ON PRICE PER BBL. OF 30° BAUME
TEXAS - LOUISIANA GULF COAST
Y E A R S
TORSION BALANCE CREWS
NO CREWS REPORTED
S.W. TEXAS DATA MISSING
TORSION BALANCE CONTRACTORS
TORSION BALANCE COMPANY-OWNED
TOTAL TORSION BALANCE CREWS
REFLECTION SEISMOGRAPH CREWS
SEISMOGRAPH CONTRACTORS
SEISMOGRAPH COMPANY-OWNED
TOTAL SEISMOGRAPH CREWS
NO CREWS REPORTED
TEXAS - LOUISIANA COAST TORSION BALANCE & SEISMOGRAPH EXPLORATIONS

FIG. 1

Further, deliberate attempts are frequently made to lead the scouts astray, and since the reporting of geophysical activities is only one field function of the company scouts in this area, they are to be congratulated that the data are no worse than they appear to be. In checking some hundred geophysical crews each week, mistakes are bound to occur. As a result we wish to thank Mr. Elton Rhine of the Empire Oil and Gas Company at Houston, Texas, for the information and reports that he has furnished us, and to congratulate him and his fellow scouts on doing a pretty good job in the face of material difficulties.

THE REFRACTION SEISMOGRAPH

Reports were available back to July 1930, so that we were able to get the fade out of the refraction technique. After plotting the price of oil over the period in question, we were justified in agreeing with the impression previously held by the senior author, viz., that the refraction technique was dropped because the law of diminishing returns ran its full course, and not that the 1931 decrease in the price of oil delivered the death blow. Just as Hobbs was not drilled on magnetic data because of the adverse results from the drilling of the three preceding magnetometer prospects, one outstanding refraction prospect on the Gulf Coast, later proved to be a large oil field by a wildcatter, was probably never drilled because of the three dry holes drilled just previously on three other refraction prospects. East Texas, however, effectively administered the *coup de grace*, and the dying gladiator never recovered. While the method was a new broom, however, it swept clean and may have passed up only one shallow or moderately deep salt dome. Once the method had shot its wad, however, it was through, and East Texas helped make that fact apparent.

THE TORSION BALANCE

We regret that data back to 1924 or 1925 are not available on torsion balance activity, for the curve back of 1930 would be most interesting. It would show a low corresponding to the point where the gravity maxima finally proved disappointing, and when the importance of gravity minima had not yet been appreciated.

From the data at hand, however, it is of interest to notice how the decrease in total torsion balance activities from late 1930 to the middle of 1931 follows the drop in the price of oil during the same period, but after that date an effect due to the price of oil does not seem apparent, for the 1933 drop in the price of oil is not reflected on

the torsion balance activity chart. However, at the first of 1933 a change from zero increase to a definite increase in company owned torsion balance activity seems to coincide with the initial production at Livingston, a gravity prospect. Paradoxically, Livingston was almost the last of the unassisted gravity discoveries, being followed only by Garwood and Cleveland. Production at Cleveland in late 1933 seems to coincide with an increase in contract torsion balance activity, carrying up to the peak which was reached and passed by late 1934.

About the time of the discovery of production at Hastings, the recognition became prevalent that torsion balance prospecting pointed the way for the seismograph, not the drill. As a result, the conclusion was drawn that isolated torsion balance surveys were of little or no practical value, and by common consent the bulk of torsion balance exploration has been left to the larger operators who can see their ways clear to mapping county by county, thereby turning up submarginal gravity prospects for later evaluation by the seismograph. As a result, contract torsion balance activity has reached the vanishing point, for although high grade equipment has been on the market for some time, many of the men who gained material torsion balance experience as contractors have been drafted by the large operators of their own crews.

Any revival of torsion balance contracting is practically out of the question, for those companies who do not have a material background of experience and data are hardly justified, in fact, there is not time enough remaining, to duplicate the extensive net works already compiled by the Humble, Shell, Gulf, Magnolia, Texas and Stanolind. However, the availability of a fieldworthy gravity meter might tempt some of the other operators into an extensive program, for the rate of coverage would be materially increased thereby, at a reduction in cost which would probably be considerable.

THE REFLECTION SEISMOGRAPH

Although the reflection seismograph had already been introduced into this province in late 1930, the price of oil was falling, and exploration was decreasing in popularity. The initial success of the method followed its use in the detail of the Iowa prospect, then recognized as a torsion balance minimum, and as something better than a marginal refraction prospect. The first well drilled was located on the reflection apex, and fortunately produced, for there seems to be no well developed central graben.

Other successes were largely in the condemning of prospects originated by other methods, and several dry holes confirmed early predictions of the reflection seismograph. Though there was little or no demand for such a method, still an excellent foundation was laid for its future use, for since the recognition of the possibility of mapping by dips had been made at Darrow, the idea was slowly but generally becoming prevalent that this was a workable method.

When the importance of the Conroe field was realized, there was large immigration of Oklahoma oil operators into the coast. Many of them were familiar with the recent and spectacular successes there of the reflection method as introduced by the Geophysical Research Corporation. These newcomers did not show the sales resistance that many of the resident companies did, and thereby the introduction of the method was facilitated. Note, however, that some company owned reflection operations had begun in 1930, and credit must be given to the Humble for having appreciated the possibilities of the method at such an early date.

After Conroe, reflection methods were generally adopted, and the total operations increased at an average rate of fifteen crews per year until at present there are some seventy-five crews, all told, operating in this province.

It is interesting to note that the growth of company operations was not affected by the 1933 price cut, while the contract activities show nearly a fifty percent decrease at the same time. Note also that the company owned operations show a remarkably steady increase each year, while the contract operations are nowhere near as regular. Further, the average (total) annual increase is made up of a company owned increase of not quite ten per year, leaving a contract increase of less than seven per year, but this is by no means a steady increase.

The material increase of total operations in late 1935 occurred largely in Chambers, Jefferson and Orange Counties (see regional activity report for Southeast Texas and South Louisiana, Geophysics, Vol. I, No. 1, Jan., 1936) and seems to have been due to the discovery of oil at North Anahuac, and the recognition of the importance of faulting. There is some suggestion that the usual fall off, after this peak, was eliminated by the crude oil price increase at the start of 1936.

The usual exploration history of any area shows a distinct over run of prospecting, a final phase of hysterical exploration running past the peak of discovery. California is an excellent example of this

determination to exhaust every possibility of finding structure. However, there was not time enough there to build up appreciable company owned operations before this over run took place, so that there the contractors enjoyed a period of great activity.

However, as the chart shows for the Gulf Coast, company owned operations in reflection exploration were initiated shortly after the introduction of the method by the contractors, and have grown at a rate greater than have the contract operations. Of the eleven largest users of the method on the Gulf Coast during 1935, the two largest were company owned. As a matter of fact, of the nine largest users, only one entirely, and only two partially, relied upon contract operations. There has been a distinct trend towards company operation, the last major defection from the contract using ranks occurring when the Stanolind bought the Western Geophysical Corporation a year or two back.

In view of this, and with the torsion balance statistics in mind, it is quite probable that the reflection seismograph over run in the Gulf Coast will be made largely by company owned crews. At the present rate it is reasonable to predict that 1937 or 1938 will be the peak year for contract operations in this province.

In conclusion, it might be well to examine the validity of this trend towards company owned operations. In general, it is based upon the following ideas.

1. By having the work fully organized within the limits of one organization, smoother control will be effected, and therefore, less opportunities for leakage of important information.

2. The contract companies, more or less openly, are interested in finding oil fields of their own, thereby competing with their clients.

In regard to the first reason, we point out that a large company organization is no bar to leaks, and are of the opinion that, on the whole, the leaks within company organizations have added up to materially more embarrassment than have those leaks within contract organizations. In regard to the second, we point out that, as serious competitors in this game of finding oil fields, the total interest plays of all the contracting companies do not result in anywhere near the competition furnished by any one of the larger users, Shell, Humble, Gulf, etc. Under the circumstances, the second reason, although one which has been advanced in all seriousness, is much the case of choking on a gnat and swallowing a camel.

Of course, adding a group of one, two or three reflection crews to

an oil company is, apparently, a relatively simple job these days. Equipment capable of recording reflections has been on the market for some time, and the contracting companies have many men, capable of operating this equipment, who could be made available at the necessary salaries. In fact, this idea has been carried to the point where equipment is available at very nominal rentals without any restrictions on the interpretation of the data secured by observers who record under specific instructions, and do nothing more.

However, there is something to be said for many contracting companies. Some of them go to extremes in order to turn out work of high caliber. The worst that can be said of these companies is that they may take an over optimistic attitude in their desire to avoid passing up oil fields. At that, perhaps, the dry hole record of some of these contracting companies is no worse than that of company owned organizations, and sometimes their record of discoveries is more impressive. These contractors, in general, do a volume of business which is large enough to justify the maintenance of appreciable and frequently capable research staffs, a function which, as Dr. Weatherby pointed out in his paper on "The Organization of an Effective Exploration Department"[2], they should carry out.

Dr. Weatherby also pointed out how a contract organization ties into a properly organized exploration department. It is frequently true that some contract companies are better organized, and often better staffed than some of the competing company owned organizations. Therefore, our suggestion is that before the executives of any company act on a decision to tack on a few reflection crews and so drop the contractors, they should examine carefully the existing organization of their own exploration department, and then consider again, that in view of the continuing progress in the art, there is probably a minimum number of reflection crews which can be operated in order to justify the organization and maintenance of the required research and auxiliary personnel.

[2] Geophysics, Vol. 1, No. 2, June 1936, pp. 179–188.

EFFECT OF DEVELOPMENT TIME AND DEVELOPER TEMPERATURE ON THE PRODUCTION OF PHOTOGRAPHIC SEISMOGRAPH RECORDS

FREDERICK A. TOMPKINS[1]

ABSTRACT

The effect of variation of development time and developer temperature on the production of uniform records is discussed, and a method of determining the relation is shown. Graphs plotted from sensitometric data indicate the need of some means of temperature control.

In a previous paper by the author,[2] some of the fundamental factors influencing the photographic processing of seismograph records were briefly outlined. Following that presentation, the desire was expressed for a more detailed and graphical presentation of some of these relations. In this paper the most important relation of development time and developer temperature to effective contrast will be discussed.

This relation can be determined most accurately by sensitometric methods. If a strip of photographic paper be exposed to a constant light source for a series of increasing exposure times, a graded series of silver deposits is secured on development. The density of these deposits can readily be measured, and the values can be expressed mathematically by the relation

$$D = \log 1/R$$

where D = density or blackness of the deposit and R = per cent of the incident light reflected from the silver deposit. On plotting these density values against the logarithm of the corresponding exposure values, the typical characteristic curve of a photographic material is secured (Fig. 1).

Now by developing several identically exposed strips for increasing times at a given developer temperature, and then by plotting the measured densities, a family of curves, Fig. 2, is secured. Such a family of curves shows the rate of density increase as a function of development time.

It will be noticed that the maximum density of this sensitive ma-

[1] Paper Manufacturing Division—Eastman Kodak Co.

[2] Fundamental Photographic Processing Operations Influencing Production of Seismograph Records. Geophysics, Vol. 1, No. 1, Page 107, Jan. 1936.

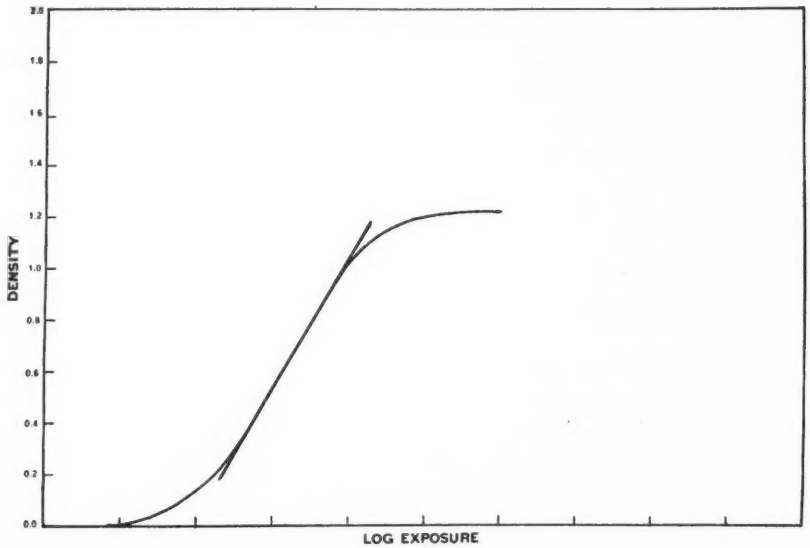

FIG. 1. Typical curve showing the relation between density and log exposure.

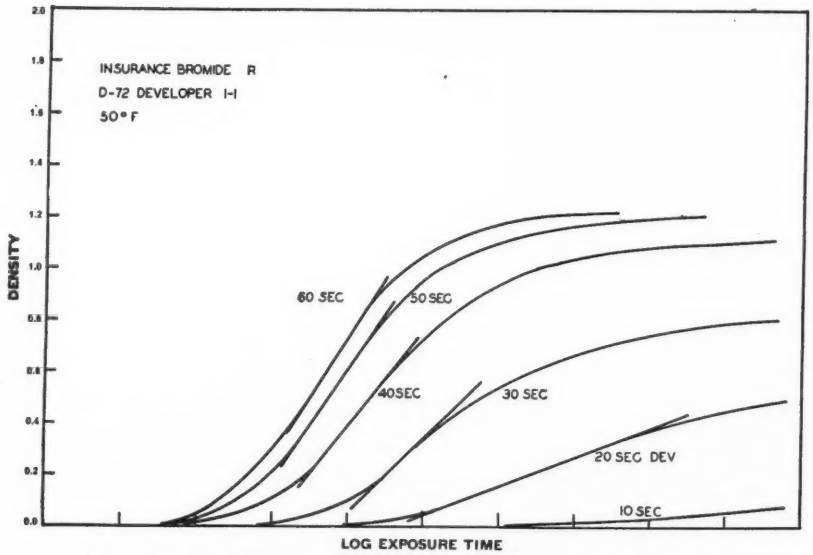

FIG. 2. Curves showing the rate of density increase as a function of development time at a given developer temperature.

terial is not attained until the development time approaches 60 seconds when development is carried out at 50°F. At 70°F. (Fig. 3)

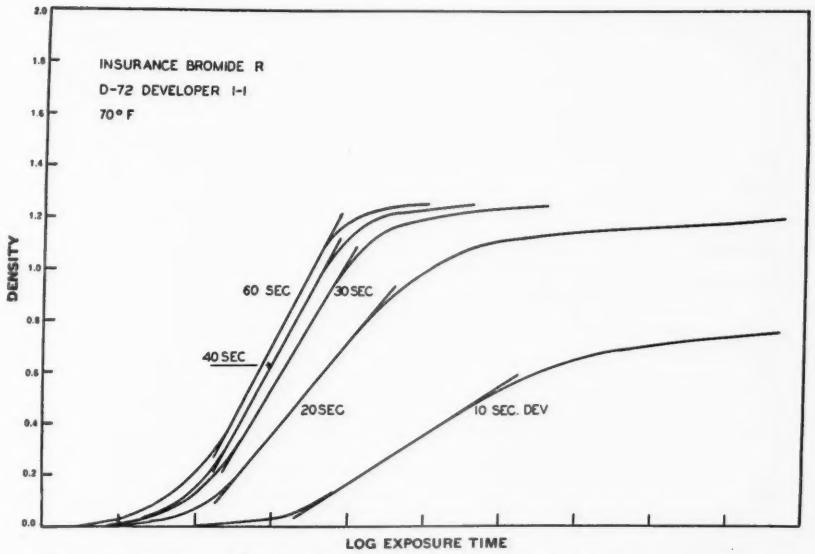

FIG. 3. Compare with Fig. 2 to note effect of raising developer temperature.

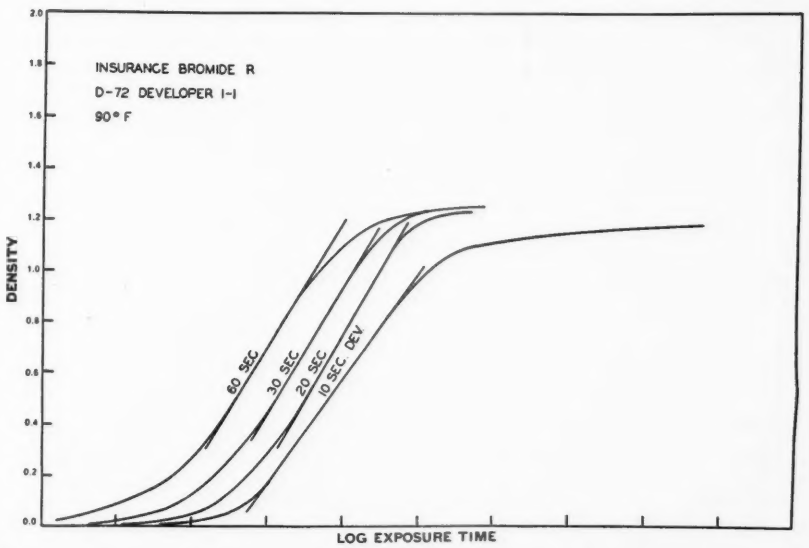

FIG. 4. Compare with Figs. 2 and 3. Note appearance of fog at 60 sec. development as indicated by the lower part of the curve lifting from the zero density line.

the maximum density is approached at 30 seconds, and at 90°F. (Fig. 4) this value is below 20 seconds. But, if the development time

is prolonged to 60 seconds, an over all greyish deposit or fog is produced. With the developer at 110°F., development is completed under ten seconds, for at this time the maximum density has been obtained, and the fog value has become pronounced. (Fig 5). Note how rapidly the fog density increases with increased development at this temperature.

Hence, it will immediately be seen that such an increase of fog produces a degraded image quality. Accordingly, if we subtract the minimum density from the maximum density (D max.$-D$ min.) for

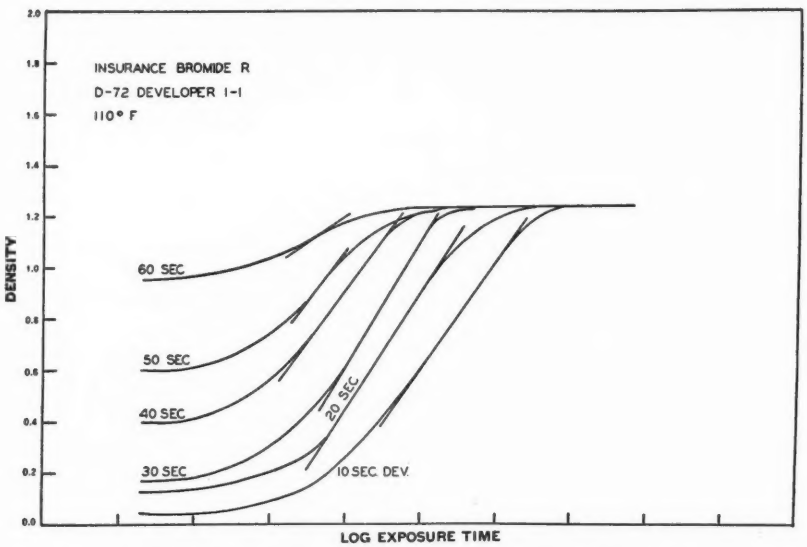

FIG. 5. Compare with Figs. 2, 3 and 4. Observe the rapid increase of fog as development time is increased at this high developer temperature.

each curve of these several families, we secure a numerical value of the effective contrast of the image in each case. The term, effective contrast, is used to denote the optimum condition of blackest black with the whitest white obtainable under each development condition. In short, it is a numerical valuation of the best image that the operator can produce under a given set of conditions.

Again, if we plot these values of effective contrast as a function of development time for each developer temperature, a series of curves (Fig. 6) is obtained which indicates the rate of change of effective contrast and which begins to show the relation of time and temperature to the final result.

Carrying the reasoning one step farther, if the development time is determined from the curves of Fig. 6, for any constant value of

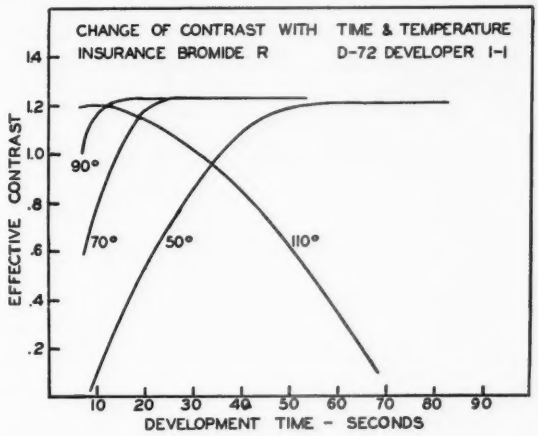

FIG. 6

effective contrast, and if these development times are plotted against the developer temperature, the final time-temperature relation becomes known. Fig. 7 shows the relation for Insurance Bromide Grade R, and Fig. 8 depicts the relation for Recording 809 when developed

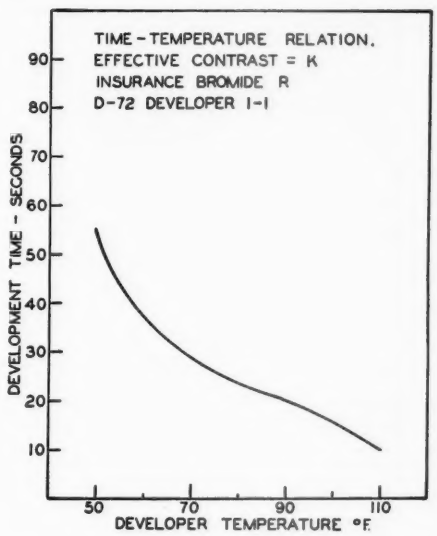

FIG. 7

in D–72 diluted 1–1. Both papers are commonly used in seismograph recording work.

Both curves indicate the great need for adjustment of development time in order to produce constant results over a wide range of temperature—a range not uncommonly found in actual field work. More important, however, they indicate the need for some means of temperature control, for it becomes practically impossible to control development at the higher temperatures, while the prolonged time

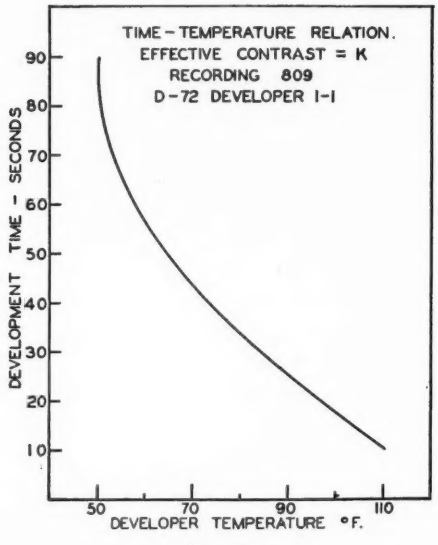

FIG. 8

necessary to produce satisfactory images at the lower temperatures may retard field operations unnecessarily.

The relations shown are for fresh developer. Used developer shifts the curve vertically along the time axis, at the same time increasing its steepness in proportion to the degree of exhaustion.

SUMMARY

The method of determining the development time-developer temperature relationship for two recording papers has been discussed, and graphs indicating such relationships are shown. These graphs indicate the need for temperature control. It is suggested that these curves can be used as the basis for determining actual operating procedure.

THE AMPLITUDE AND CHARACTER OF REFRACTION WAVES

A. WOLF*

ABSTRACT

In seismic exploration it is assumed that elastic waves can be refracted into and out of a high speed medium at the angle of total reflection and meanwhile travel along the interface between the two media. That such a path is in agreement with the general theory of elastic waves has been proved by Muskat. The present paper is an extension of the work of Muskat, and shows in addition that the general theory can explain the low amplitude and low frequencies of these refracted waves, and is otherwise in agreement with observation.

Equations are developed for the effect at any point of the low speed medium of an impulse or of a periodic disturbance initiated at any other point of the same medium. Details are worked out for the case where the recorder is so situated that the refracted wave arrives first. The direction of the arriving wave is found to be that of the angle of total reflection, and its amplitude to diminish with the square of the distance from the shot point—in agreement with Muskat. The amplitude of the refracted wave is also found to be only a small fraction of that of the direct wave, or of that of the reflected wave which arrives still later. The motion produced by the refracted wave is found to be a unidirectional pulse which lasts until the arrival of the direct wave.

Finally, the appearance of a record of motion is given for certain assumed numerical data. For this case it is found that the average amplitude of the refracted wave is 8 per cent of that of the direct wave.

It is generally assumed in seismic exploration that elastic waves can be refracted both into and out of a high speed medium at the angle

FIG. 1

of total reflection, to be recorded on the surface of the ground. A path, such as shown in Fig. 1, has become associated with this work. The object of the present paper is to show that the general theory of elastic waves accounts for the existence of that type of propagation and is in general agreement with observation. It will be possible to explain both the low amplitude and the relatively low frequencies of these refracted waves.

Perhaps the first paper to call attention to the application of the

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exact equations to this subject was published by Muskat.[1] He used a method well known from the work of Sommerfeld on the propagation of electromagnetic waves. Muskat's work proves that the type of path assumed in seismology for refraction waves really exists, but his conclusions about the order of magnitude of the amplitude of such waves seem to be incorrect, and his discussion gives very little idea of their character.

The present discussion follows the methods developed in a paper of H. Lamb: "On the Propagation of Tremors over the Surface of an Elastic Solid."[2]

Assume that there are two elastic solids A and B extending to infinity in all directions from a plane of separation SS between them (cf. Fig. 2).[3] The reciprocals of the velocities of propagation of

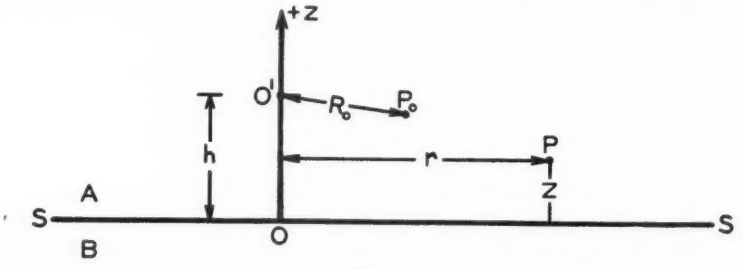


FIG. 2

longitudinal waves in A and B will be denoted by a and b respectively. It is assumed that a is greater than b, which means that the velocity in A is smaller. Writing

$$a/b=n$$

it follows $n>1$.

Let the densities of A and B be respectively γ_A and γ_B. A system of cylindrical coordinates (r, z) will be used whose origin, O, is located on SS. The positive direction of the z axis is in A. At a distance h above O there is a small cavity O' in which a shot, which is supposed to be radially symmetrical, is set off at the time $t=O$. At a point P_o, a distance R_o away from O', there is observed the elastic displacement $u_o(t)$. According to the hypothesis made regarding the nature of the shot, this displacement is directed along the line $O'P_o$. In what follows the function $u_o(t)$ will be referred to as the impulse. The prob-

[1] Muskat: Physics 4, 1933, p. 14.

[2] Lamb: Phil. Trans. Roy. Soc. 203, 1904, p. 31.

[3] The effects produced by the surface of the ground are not considered in this paper.

lem is to determine the horizontal and vertical elastic displacements v, w, at a point P (coordinates r, z) in A, which correspond to the impulse $u_o(t)$. It is somewhat simpler to study the refraction as a function of $u_o(t)$ than it would be to derive the corresponding equations if the pressure in the cavity O' were given as a function of time.

The procedure can be outlined as follows: First the expression for the elastic potential of longitudinal waves ϕ is determined on the assumption that the impulse is a periodic function of time

$$u_0(t) = u_0 e^{j\omega(t-aR_0)}.$$

By the use of the Fourier integral an equation for ϕ is then obtained which corresponds to an impulse of infinitesimal duration (cf. Fig. 3). By superposing such solutions it is easy to determine ϕ, v, w for any impulse $u_o(t)$.

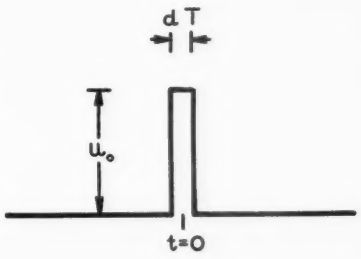


FIG. 3

In the case of a periodic impulse, it is easy to show that

$$\sigma\phi = -\frac{u_0 R_0}{\pi}\int_{-\infty}^{\infty}\frac{\lambda d\lambda}{\sqrt{\lambda^2-1}}f(\lambda)\int_0^{\infty}dx. \tag{1}$$
$$\exp j\omega[t-\sqrt{1-\lambda^2}a(z+h)-\lambda ar\cosh x]$$

where $f(\lambda)$ is the coefficient of reflection of plane longitudinal waves at an angle φ given by the equation

$$\sin\varphi=\lambda$$

and σ is the operator

$$1+\frac{1}{aR_0}\int_0^t dt.$$

Except for minor changes, Equation (1) is the same as Muskat's equation for the potential. The Bessel function which he uses is replaced by an integration with respect to x in Equation (1). The method of deriving such equations is well known from Lamb's work on Rayleigh waves and will, therefore, be omitted. The function

$f(\lambda)$ depends on the Poisson's ratio of the solids and is very complicated. If, however, the transformation of longitudinal waves into transverse is neglected, the expression

$$f(\lambda) = \frac{\gamma_B\sqrt{\lambda^2 - 1} - \gamma_A\sqrt{\lambda^2 - 1/n^2}}{\gamma_B\sqrt{\lambda^2 - 1} + \gamma_A\sqrt{\lambda^2 - 1/n^2}} \tag{2}$$

is obtained. In what follows $f(\lambda)$ will be assumed to have this simple form and furthermore $\gamma_A = \gamma_B$. It is possible to calculate numerically the coefficients of reflection is special cases, and show that no great differences exist between the exact values of the reflection coefficient and form given by Equation (2), at least in the range of values of λ which is relevant. The following equations could, to a great extent, be written out with $f(\lambda)$ undetermined, and only the final calculation requires its knowledge. This makes it possible to state that no qualitative differences exist between the conclusions reached in this paper and anything which could be obtained by using more exact expressions.

The integrand in Equation (1) can be regarded as a function of the complex variable λ, provided, only, care is taken to use a suitable path of integration in the λ-plane. Such a path is shown in Fig. 4a. The square roots must be taken positive for $\lambda = +\infty$. Since the integrand vanishes exponentially at infinity in the lower half-plane, it is possible to modify the path of integration of Fig. 4a into that shown in Fig. 4b, which is chosen so as to make all subsequent integrations convergent. It will be noticed that the point

$$\alpha = \frac{r}{\sqrt{r^2 + (z + h)^2}}$$

is shown between $1/n$ and 1. This means that the recording station P, is located in such a way that the refracted wave reaches it before the direct, which is obviously the only interesting case. If transverse waves were taken into consideration, the contour of integration would have to be modified only slightly. The most important change would be the addition of small circles around the poles of $f(\lambda)$ which correspond to waves similar to Rayleigh Waves propagated along the interface of A and B.

The refracted waves are given by those parts of the contour of integration which are between $1/n$ and α. The potential is given by

$$\sigma\phi_r = -\frac{u_0 R_0}{\pi}\int_{1/n}^{\alpha}\frac{\lambda d\lambda}{\sqrt{\lambda^2-1}}Imf(\lambda)\int_0^{\infty}dx. \tag{3}$$
$$\exp j\omega[t-\sqrt{1-\lambda^2}a(z+h)-\lambda ar\cosh x].$$

Multiplying the right hand side by $d\omega$ and integrating from O to ∞, the potential corresponding to an impulse of short duration dT and

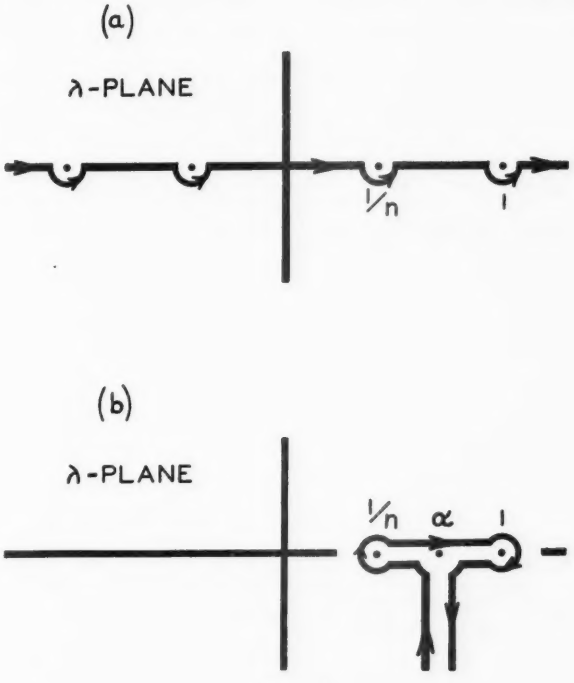


FIG. 4[4]

amplitide u_o (cf. Fig. 3) is obtained. Since the quantities between the square brackets are real, the result of an integration of the exponential with respect to ω is

$$\delta[t-\sqrt{1-\lambda^2}a(z+h)-\lambda ar\cosh x]$$

where the function δ is defined by the equations

$$\delta(y) = 1 \quad \text{for} \quad y^2 < \left(\frac{dT}{2}\right)^2$$

[4] The half circles around 1/n and 1 in Fig. 4a should be on the upper side of the real axis.

$$\delta(y) = 0 \quad \text{for} \quad y^2 > \left(\frac{dT}{2}\right)^2.$$

After this has been done, it is easy to carry out the integration with respect to x and obtain a relatively simple equation for the elastic potential. Differentiating it with respect to r and z gives the elastic displacements

$$(4)\quad \begin{aligned} \sigma v &= \frac{4adT}{1 - 1/n^2}\,\frac{u_0R_0}{\pi} \\ &\frac{1}{an}\,\frac{d}{dt}\int_{1/n}^{\lambda_0} \lambda\sqrt{1-\lambda^2}\,\frac{t - \sqrt{1-\lambda^2}a(z+h)}{\sqrt{[t-\sqrt{1-\lambda^2}a(z+h)]^2 - \lambda^2a^2r^2}}\,d\lambda \\ \sigma w &= \frac{4adT}{1 - 1/n^2}\,\frac{u_0R_0}{\pi} \\ &\frac{d}{dt}\int_{1/n}^{\lambda_0} \lambda\sqrt{1-\lambda^2}\sqrt{\lambda^2 - \frac{1}{n^2}}\,\frac{d\lambda}{\sqrt{[t-\sqrt{1-\lambda^2}a(z+h)]^2 - \lambda^2a^2r^2}} \end{aligned}$$

for

$$t > t_0 = \sqrt{1 - 1/n^2}a(z + h) + \frac{1}{n}\,ar$$

and $v=w=0$ for $t<t_0$.
λ_0 is a parameter such that

$$1/n < \lambda_0 < 1 \quad \text{and} \quad t = \sqrt{1 - \lambda_0^2}a(z + h) + \lambda_0 ar$$

t_0 is obviously the time at which the refracted wave reaches the recording point P. It is, of course, in agreement with calculations made on the basis of the minimum time principle.

The evaluation of the integrals is much simplified by the assumption

$$z+h=0$$

which means that both the shot point and the recording apparatus are located close to the surface of separation of A and B. The results must, apart from details, be true when $(x+h)/r<<1$.

The following expressions are obtained in this case

$$(5)\quad \begin{aligned} \sigma v &= \frac{dT}{1 - 1/n^2}\,u_0R_0\,\frac{1}{ar^2}\left(\frac{3t^2}{a^2r^2} - \frac{1}{n^2}\right) \\ \sigma w &= \frac{4dT}{\sqrt{1 - 1/n^2}}\,\frac{u_0R_0}{\pi ar^2}\,\frac{t}{ar}\,[2E(k) - F(1R)] \end{aligned}$$

$$\text{for } t > \frac{1}{n} ar \quad \text{and} \quad v = w = 0 \quad \text{for } t < \frac{1}{n} ar$$

where

$$k^2 = \frac{t^2/a^2r^2 - 1/n^2}{1 - 1/n^2}$$

and $F(k)$ and $E(k)$ are respectively the complete elliptic integrals of the first and second kind. It follows that, at the time of arrival of the wave, the direction of motion makes an angle with the normal to the surface of separation SS between the two solids, which is equal to the angle of total reflection.

It is seen also that the amplitude of motion decreases as the square of the distance from the shot point; this is in agreement with the results obtained by Muskat.

It remains to explain the nature of the operator $\sigma = 1 + \frac{1}{aR_0p}$ and to generalize the results for the case of pulses of finite duration. First, as regards the operator, it is possible to show that, if P_o is not too close to the shot point and the time is not much greater than aR_o, the operator can be neglected. For any other case, the operator can be evaluated according to rules of the operational calculus.

It is very easy to generalize Equation (5) for the case of impulses of finite duration. They are first rewritten in the form

$$v = \frac{R_0}{ar^2} f(t) u_0 dT$$

$$w = \frac{R_0}{ar^2} g(t) u_0 dT.^5$$

If the actual displacement at R_o is $u_o(t)$ instead of $u_o\delta(t - aR_o)$,

$$v = \frac{R_0}{ar^2} \int_0^t u_0(x) f(t - x) dx$$

$$w = \frac{R_0}{ar^2} \int_0^t u_0(x) g(t - x) dx.$$

Let it next be supposed that the impulse is confined to a short, though not infinitesimal, time T_o. Then, approximately

[5] f, g are of the order of magnitude of 1.

$$v = \frac{R_0}{ar^2} f(t) \int_0^{T_0} u_0(x)dx$$

$$w = \frac{R_0}{ar^2} g(t) \int_0^{T_0} u_0(x)dx.$$

The character of the motion produced by the refracted wave is seen to be independent of the shape of the impulse and consist of a long pulse occupying the whole interval of time between the first arrival of the refracted wave and the arrival of the direct wave. Aside from a numerical factor of the order unity, the ratio of the amplitudes of elastic displacement of the refracted and direct waves is given by

$$\frac{1}{ar} \frac{\int_0^{T_0} u_0(x)dx}{u_0 \text{ (max)}}.$$

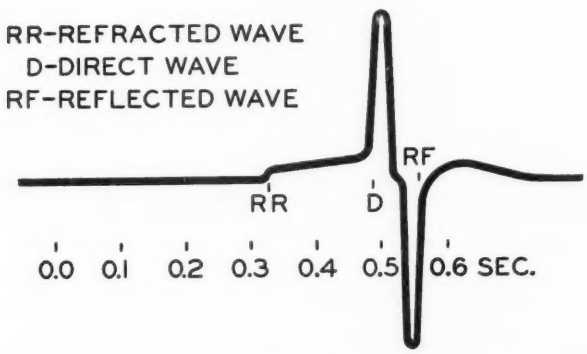


FIG. 5

In conclusion, the appearance of a record of motion will be discussed for a case in which $T_0 = 0.02$ sec., $n = 1.5$, $1/a = 10{,}000$ ft/sec., $r = 5000$ ft. A sketch of the record of horizontal motion is shown in Fig. 5. The refracted wave arrives at the recording station at the time 0.33 sec. after the shot and consists of a unidirectional pulse which lasts until the time 0.5 sec. when the direct wave arrives. Its average amplitude is 0.08 the amplitude of the direct wave. A short time after the arrival of the direct wave a reflection from the interface is observed. Since the angle of incidence is assumed to be very nearly grazing, the direction of motion of the reflection is negative.

THE SEISMIC ELECTRIC EFFECT[1]

R. R. THOMPSON[2]

ABSTRACT

The seismic electric effect is the name which has been given to the variation of earth resistivity with elastic deformation. An account is given of the use of this effect in recording seismic waves. The recording circuit employs a direct current sent through the ground by means of a storage battery and spaced electrodes, and the changes in current due to seismic disturbances are recorded. An analysis of the noise appearing in the recording circuit is given and methods of reducing the noise level are described. The results of measurements on the magnitude of the effect are given, and the sensitivity obtainable with the seismic electric method is compared with that given by an electromagnetic pickup.

The possibility that there might exist a variation of earth resistivity with elastic deformation was first suggested by Louis Statham and L. W. Blau of the Humble Company. They pointed out that, if the effect existed, it might be used to provide an entirely new method of picking up the seismic energy in reflection and refraction shooting. By a suitable arrangement of electrodes it would be possible to obtain a current distribution in the earth involving almost any desired volume of earth. Since the resistivity at any point in this volume would vary, as a function of time, with the elastic strain at that point, the variation in effective impedance between the two groups of electrodes would be the integral, throughout the volume, of the seismic disturbance as a function of time. Due to the very high apparent velocity (along the surface) of the reflected waves as compared to waves of less penetration, the method would offer a possibility of discriminating against the shallow waves and in favor of the reflections.

This effect, the variation of the earth resistivity with elastic deformation, which has been termed the seismic-electric effect, has been investigated as to causes, magnitude and possibilities of use.

A schematic diagram of the circuit used in recording the seismic electric effect is shown in Fig. 1A. For simplicity, only two electrodes *A* and *B* are shown, although either of these may be a multiple electrode earth contact. A direct current is passed through the earth between these electrodes by means of a storage battery in series with the primary of a transformer. The secondary of the transformer is connected through an amplifier to an oscillograph and the signals are

[1] Published by permission of the Board of Directors, Humble Oil & Refining Company.

[2] Geophysics Department, Humble Oil & Refining Company.

recorded as in ordinary reflection shooting. Fig. 1B represents the equivalent input circuit in which R represents the effective ground resistance, Z is the impedance looking into the transformer primary, and I is the direct current flowing in the circuit. The resistance R is assumed to vary with time as

$$R = R_0 + \Delta r(t)$$

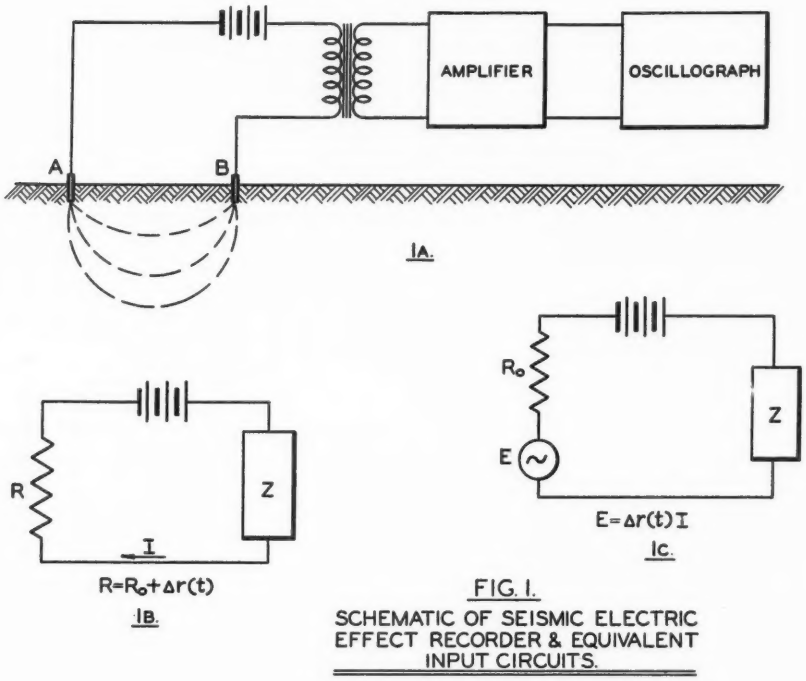


FIG. 1.
SCHEMATIC OF SEISMIC ELECTRIC EFFECT RECORDER & EQUIVALENT INPUT CIRCUITS.

where $\Delta r(t)$ is very small compared to R_0. This circuit is approximately equivalent to that of Fig. 1C where the effect of the variation in resistance is represented by a series generator of voltage $\Delta r(t)I$ and internal impedance R_0. Thus the signal voltage effective in the input circuit is given by the product of the instantaneous value of the resistance variation and the magnitude of the direct current flowing through the ground.

Noise voltages which originate in the input circuit may be divided into two classes: Noises due to resistance changes other than the seismic electric effect, such as variations in contact resistance; and noises due to currents already flowing in the earth which produce a fluctuating potential difference between the electrodes. The first type

of noise increases linearly with the ground current so long as the current is not raised to such a high value that the unwanted resistance fluctuations cease to be independent of the current. The second type of noise voltage is independent of the ground current and is a function only of the location, separation, etc., of the electrodes. In addition, it has been found that for large values of current a third noise voltage component must be included. This component is negligible for small values of current but increases rapidly as the current is raised above a certain value. Apparently this source of noise is associated with chemical action at the electrode.

Thus an expression for the noise voltage may be written

$$E_n = IR_n + E_g + G(I)$$

where I is the ground current, R_n represents resistance fluctuations other than those due to the seismic electric effect, E_g represents the noise voltage picked up due to natural or artificial earth currents, and $G(I)$ is a term included to take care of the very rapid increase of noise with ground current when the current becomes too high.

These noise components are illustrated in Fig. 2 where it is seen that at very low currents E_g is the important component; as the current is increased IR_n becomes the important component; and for very large values of current $G(I)$ becomes the principal source of noise. Since the signal voltage varies linearly with the ground current, that is

$$E_s = \Delta r I$$

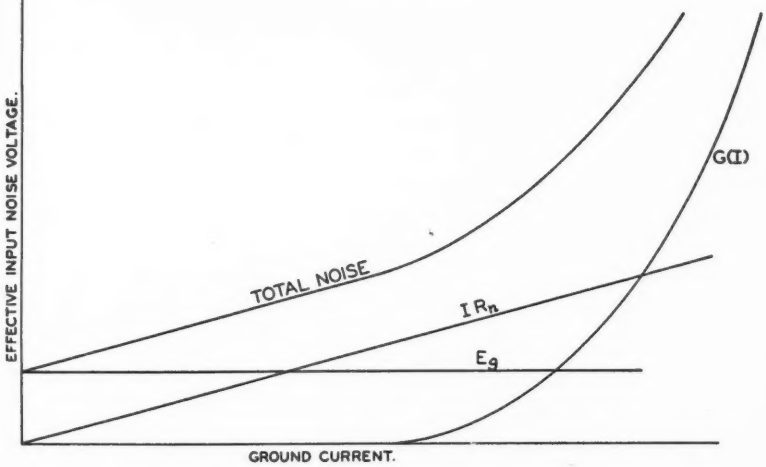
it is evident that the best signal to noise ratio is obtained in the region where the IR_n term is the principal noise component and that in this region the ratio is

$$\frac{E_s}{E_n} = \frac{\Delta r}{R_n}.$$

The first experiments with electrodes showed that any kind of metal rods driven into or buried in the earth gave an intolerably high noise level due to fluctuating contact resistances. A much quieter earth contact was finally obtained by drilling holes in the ground with a hand auger, filling the holes with a salt solution to serve as an electrolyte, and placing the electrode in the solution. The electrodes used were cylinders of copper or zinc about $3\frac{1}{2}$ inches in diameter and 3 to 6 inches in length. Electrodes of this type were used in all subsequent work.

Measurements were made, with two such electrodes, on noise as a function of electrode separation. The results are shown on Fig. 3. Line A represents the equivalent input noise voltage for various distances between electrodes. It is evident that in this case the noise increases linearly with distance between electrodes. This indicates that the noise comes from ground currents which are distributed in such a way that the potential gradient is very nearly constant over the area involved. This would be expected to be the case if the earth

FIG. 2.
ILLUSTRATION OF NOISE COMPONENTS IN SEISMIC ELECTRIC RECORDER.



is homogeneous, and the source of the disturbance is not a local source near the electrodes. If this is true, it should be possible to balance out the voltage picked up in this way by using the arrangement *B*, where three electrodes are set in a straight line, the center electrode being equidistant from the two outside electrodes. The two outside electrodes are connected together and are of one polarity while the center electrode is of the opposite polarity. With an arrangement of this type the noise dropped to the value indicated by "B" on Fig. 3 when the distance between outside electrodes was 50 feet. Five electrodes in line gave a noise voltage indicated by "C" on Fig. 3. Any number of electrodes may be used in a balanced arrangement, it being necessary only to space the electrodes so that the vector sum of all the lines correcting positive electrodes to negative

electrodes is zero. It thus appears that when only two electrodes are used the E_n term is relatively large and increases linearly with electrode separation. With a balanced arrangement the E_n term can be reduced considerably.

The results of measurements on the variation of noise with ground current are shown on Fig. 4. In this case a three electrode balanced circuit was used so that the E_n term should be small, and the IR_n term should be the important one. It will be noted that the noise increases linearly with the current and has a very small value at zero

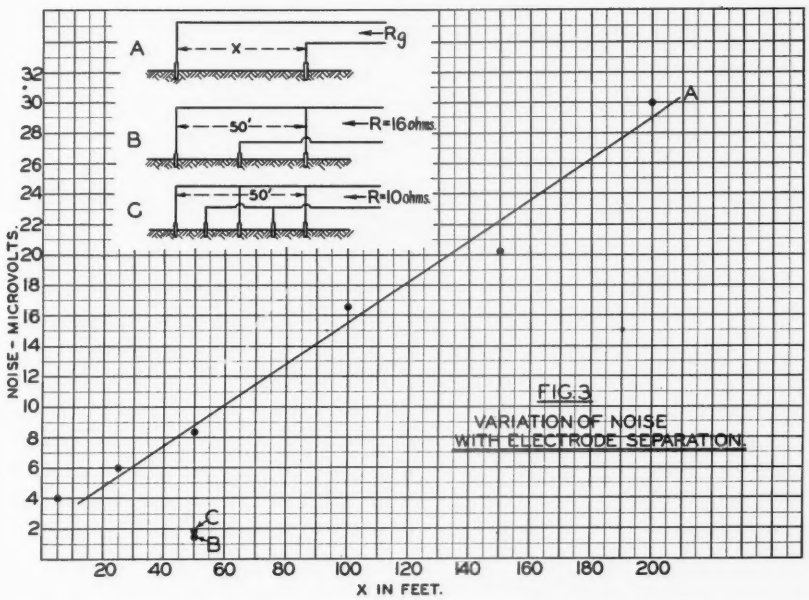


FIG. 3
VARIATION OF NOISE WITH ELECTRODE SEPARATION.

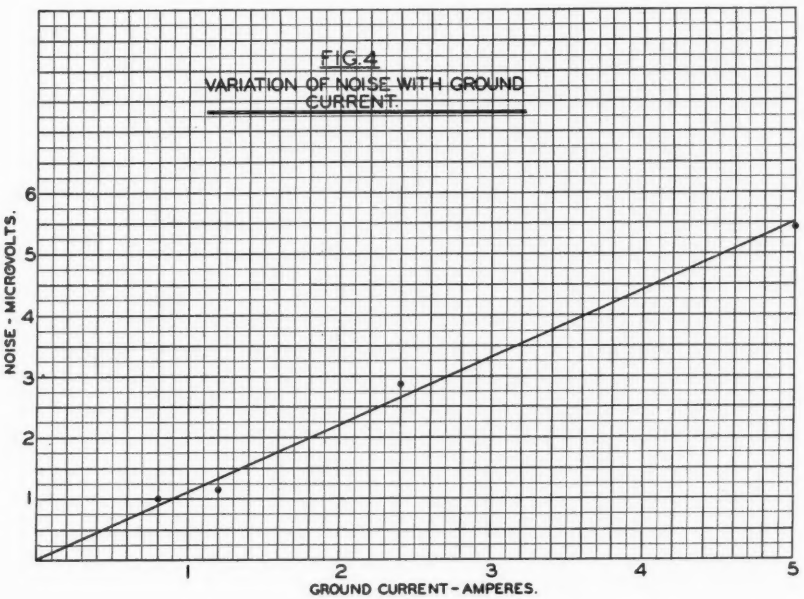
current as would be expected, since the noise at zero current should be the value of E_n.

As has already been shown, the effective input signal voltage is given by

$$E_s = \Delta r I .$$

The value of Δr, and hence of the signal voltage, varies with time in exactly the same way as the ground motion. By inserting a low impedance generator of proper frequency and known voltage in the input circuit and comparing the amplitude recorded from this input to that obtained on the record of a dynamite shot, the magnitude of E_s may be determined at various times on the record. The magnitude

of the ground current may be measured by a series ammeter and the ground resistance determined by the IR drop across the electrodes. Thus Δr and R may be determined and the ratio $\Delta r/R$ computed. This ratio is, of course, a function of time along the record, and for any particular set-up will vary with the size of the charge in the same way as the ground motion. Fig. 5 shows the values obtained for $\Delta r/R$ for a number of locations in the Gulf Coast. The values of Δr are taken from the maximum amplitudes recorded in the neighborhood of .5, 1, 1.5, and 2 seconds. It will be seen that the resistance

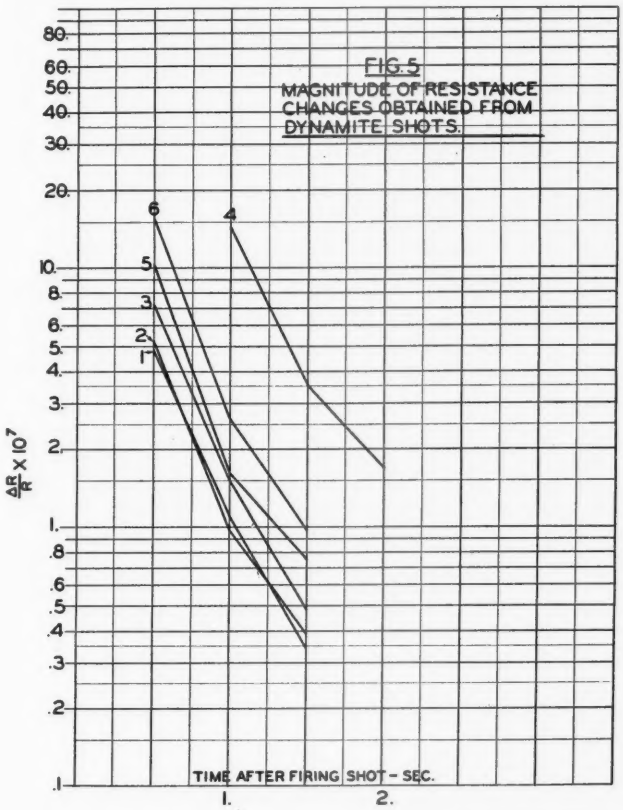


variation due to the energy arriving at about one second out on the record is of the order of 1 to 15 parts in 10 million and at two seconds it is about a tenth as much.

The charges used vary from $2\frac{1}{2}$ to 10 pounds and are of about the same size as the average charge for ordinary reflection shooting in the same locality. The distance from the shot to the recorder varies from 2000 to 2500 feet. The largest effect observed, that of No. 4, represents a usable amplitude on the record about equal to that obtained from a standard electromagnetic pickup.

Photographs of two records with seismic electric traces are shown on Fig. 6. On record No. 1 a standard pickup was placed beside the

center electrode of the seismic electric set-up which was 1900 feet from the shot; the outside electrodes were fifteen feet on either side of the center electrode. Identical amplifiers of the resistance coupled type with no low frequency elimination filters were used for the two traces. On record No. 2 the fourth trace of a standard field set-up was replaced by a seismic electric trace. The spread for this record was



2100 to 2850 feet, number three pickup was at 2400 feet and the seismic electric electrodes were at 2375, 2400, and 2425 feet. The sensitivity on these records is sufficiently high for practical field use but they represent an effect of about the magnitude given by curve 4 of Fig. 5, that is, they were taken in the most favorable areas found. In many areas a much lower sensitivity was found to exist. The problem of increasing the usable sensitivity is entirely a matter of

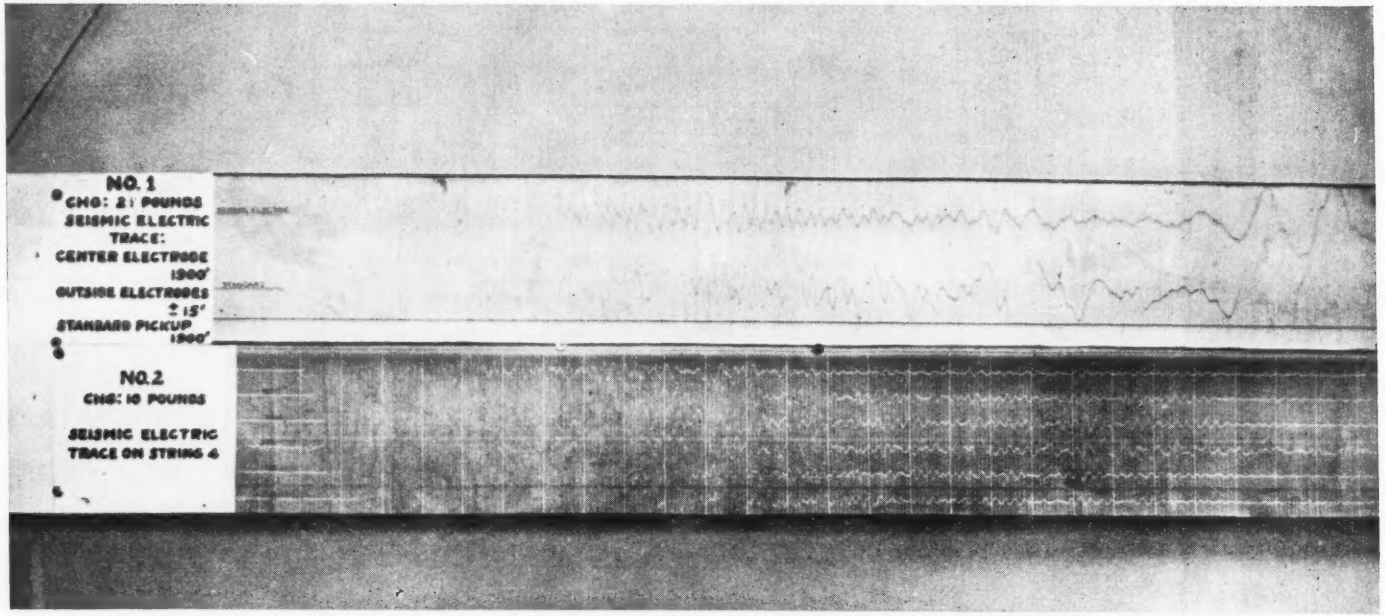


FIG. 6. Comparison of Seismic Electric Recorder Electro-magnetic pickup.

reducing the noise level and it is possible that means for accomplishing this may be found at a future date.

Since the potential gradient, due to the current sent through the earth, is large near the electrodes and drops off rapidly for increasing distances from the electrodes, the principal contributions to the input signals come from the regions near the electrodes. Consequently the seismic electric recorder does not integrate the seismic disturbance uniformly throughout the volume of earth which carries the current, but gives most weight to the disturbance near the electrodes.

As to the mechanism of the resistance change, no definite answer has been found. Experiments on electrodes set close together so that the intervening earth becomes saturated with the electrolyte show very little response, indicating that the effect is in the earth itself and not in the electrolyte or at the electrode surface. It has been suggested that the effect might be due to the small temperature fluctuations which occur with the compression in the earth and that, on account of the temperature coefficient of resistivity of the earth, a resistance fluctuation is produced. It seems unlikely, however, that if this were true, there would exist such large variations in the magnitude of the resistance change as have been observed from one location to another. A more probable explanation seems to be that the effect is some sort of loose contact phenomenon between the particles of the earth.

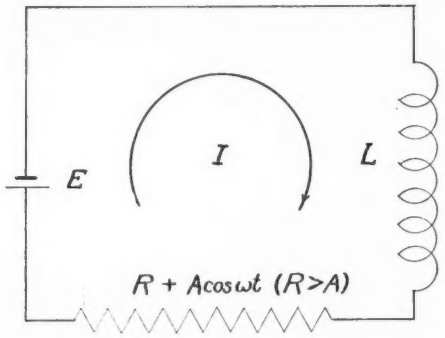
A SIMPLIFIED CIRCUIT OF THE SEISMIC ELECTRIC METHOD AND ITS STEADY-STATE SOLUTION[1]

M. M. SLOTNICK[2]

ABSTRACT

The Seismic Electric Effect gives rise to the problem of finding the steady state response of a circuit consisting of an inductance and a response of a circuit consisting of an inductance and a resistance of the form $R+A\cos\omega t$ $(R>A)$ in series with a D.C. input. In this paper a solution is given, other than the one usually obtained by the method of successive approximations.

The circuit shown in the accompanying figure was suggested by the seismic-electric method[3] as being the simplest possible one which might arise in a very ideal situation. Although the amplitude and phase of as many of the harmonics as desired can be computed to any degree of accuracy by the usual and well-known methods[4] of successive approximations, it may not be entirely without interest to indicate the solution in series form and the method by which this solution is obtained.



In the figure, L denotes a constant inductance in series with a d. c. input voltage E, and a resistance of the form $R+A\cos\omega t$, in which R and A are both constant with $R>A$. The differential equation for the current I is

[1] Presented by permission of the Board of Directors of the Humble Oil and Refining Company, Houston, Texas.

[2] Mathematician, Geophysics Department, Humble Oil and Refining Company.

[3] Cf. R. R. Thompson, The Seismic-Electric Effect and its Use in Recording Seismic Waves, Geophysics, Vol. 1, No. 3, pp. 327–335.

[4] Cf., e.g., E. A. Guillemin, Communication Networks, vol. 1, (New York, 1931), p. 403, et seq.

$$(1) \qquad (R + A\cos\omega t)I + L\frac{dI}{dt} = E,$$

and the solution, subject to the boundary condition,

$$(2) \qquad I(0) = 0$$

is

$$(3) \qquad I = ke^{-at-b\sin\omega t}\int_0^t e^{at+b\sin\omega t}dt,$$

in which we have set

$$(4) \qquad k = E/L, \qquad a = R/L, \qquad b = A/L\omega.$$

The problem, then, is to find $I(t)$ as $t\to\infty$, inasmuch as we are interested in the steady-state part of (3).

Let us assume that this steady-state part of $I(t)$ is $I^*(t)$ and that its Fourier expansion is

$$(5) \qquad I^*(t) = \sum_{n=0}^{\infty}(i_n\cos n\omega t + j_n\sin n\omega t).$$

Since $I^*(t)$ must satisfy (1) identically in t, the Fourier coefficients i_n and j_n must satisfy the equations:

$$(6) \qquad \begin{aligned} ai_0 + \tfrac{1}{2}b\omega i_1 &= k,\\ ai_1 + \omega j_1 + b\omega i_0 + \tfrac{1}{2}b\omega i_2 &= 0,\\ -\omega i_1 + aj_1 + \tfrac{1}{2}b\omega j_2 &= 0,\\ ai_n + n\omega j_n + \tfrac{1}{2}b\omega(i_{n-1} + i_{n+1}) &= 0,\\ aj_n - n\omega i_n + \tfrac{1}{2}b\omega(j_{n-1} + j_{n+1}) &= 0. \end{aligned} \quad \Big\} \; n = 2, 3, 4, \cdots$$

If, then, we succeed in finding the series (5) for $I^*(t)$, we shall also succeed at the same time in finding the solution for this infinite set of linear equations in the infinite number of unknowns i_n, j_n.

We begin with the well-known expansion[5]:

$$(7) \qquad e^{-b\sin\omega t} = J_0 + 2\sum_{n=1}^{\infty}J_{2n}\cos 2n\omega t - 2i\sum_{n=1}^{\infty}J_{2n-1}\sin(2n-1)\omega t.^{6}$$

We multiply this expression by e^{at} and integrate term by term with

[5] Cf. G. N. Watson, Theory of Bessel Functions, (Cambridge, 1922), p. 22.

[6] In what follows, and here, also, the symbol J_n shall indicate the value of the nth Bessel coefficient for the argument ib ($i=\sqrt{-1}$).

respect to t between the limits 0 and t. In this way we find that, as t is allowed to become infinite, $I(t)$ approaches

$$I^*(t) = ke^{-b\sin\omega t}\left\{\frac{J_0}{a} + 2\sum_{n=1}^{\infty} J_{2n}\,\frac{a\cos 2n\omega t + 2n\omega \sin 2n\omega t}{a^2 + (2n)^2\omega^2}\right.$$
$$\left. - 2i\sum_{n=1}^{\infty} J_{2n-1}\,\frac{a\sin(2n-1)\omega t - (2n-1)\omega\cos(2n-1)\omega t}{a^2 + (2n-1)^2\omega^2}\right\}. \tag{8}$$

The factor $e^{b\sin\omega t}$ is now replaced by an expression similar to (7) in which b is replaced by $-b$. These two series are multiplied term by term and the corresponding harmonics are regrouped with the result that we find

$$I^*(t) = k\left\{\sum_{l=-\infty}^{\infty}\frac{aJ_l^2}{a^2 + l^2w^2} + \sum_{n=1}^{\infty}\left[\left(\sum_{l=-\infty}^{\infty}\frac{2aJ_lJ_{2n+l}}{a^2 + (2n+l)^2w^2}\right)\cos 2n\omega t\right.\right.$$
$$+\left(\sum_{l=-\infty}^{\infty}\frac{2(2n+l)\omega J_lJ_{2n+l}}{a^2 + (2n+l)^2\omega^2}\right)\sin 2n\omega t$$
$$+\left(\sum_{l=-\infty}^{\infty}\frac{2iaJ_lJ_{2n+l-1}}{a^2 + (2n+l-1)^2\omega^2}\right)\sin(2n-1)\omega t \tag{9}$$
$$\left.\left.+\left(\sum_{l=-\infty}^{\infty}\frac{2i(2n+l-1)\omega J_lJ_{2n+l-1}}{a^2 + (2n+l-1)^2\omega^2}\right)\cos(2n-1)\omega t\right]\right\}.$$

That each Fourier coefficient is real is, of course, obvious. That it converges follows from the following considerations. We shall prove it for the coefficient of $\sin 2n\omega t$, neglecting the factor 2. Bearing in mind that $|J_l| = |J_{-l}|$,

$$\sum_{l=-\infty}^{\infty}\left|\frac{(2n+l)\omega J_lJ_{2n+l}}{a^2 + (2n+l)^2\omega^2}\right| < \sum_{l=-\infty}^{\infty}|J_lJ_{2n+l}|$$
$$= \sum_{l=0}^{2n}|J_lJ_{2n-l}| + \sum_{l=2n+1}^{\infty}|J_lJ_{l-2n}| + \sum_{l=0}^{\infty}|J_lJ_{2n+l}|$$
$$< e^{b^2/2(n+1)}\left\{\sum_{l=0}^{2n}\frac{\left(\frac{b}{2}\right)^l\left(\frac{b}{2}\right)^{2n-l}}{l!(2n-l)!} + \sum_{l=2n+1}^{\infty}\frac{\left(\frac{b}{2}\right)^l\left(\frac{b}{2}\right)^{l-2n}}{l!(2n+l)!}\right.$$
$$\left. + \sum_{l=0}^{\infty}\frac{\left(\frac{b}{2}\right)^l\left(\frac{b}{2}\right)^{2n+l}}{l!(2n+l)!}\right\}^{7}$$

[7] Cf. Watson, loc. cit., p. 16, (4).

$$< e^{b^2/2(n+1)} \left\{ \frac{(2n+1)}{(n!)^2}\left(\frac{b}{2}\right)^{2n} + \left(\frac{b}{2}\right)^{-2n} \sum_{l=2n+1}^{\infty} \frac{\left(\frac{b}{2}\right)^{2l}}{l!} + \frac{\left(\frac{b}{2}\right)^{2n}}{(2n)!} \sum_{l=0}^{\infty} \frac{\left(\frac{b}{2}\right)^{2l}}{l!} \right\}$$

$$< e^{b^2/2(n+1)} \left\{ \frac{(2n+1)}{(n!)^2}\left(\frac{b}{2}\right)^{2n} + \left(\frac{b}{2}\right)^{-2n} e^{b^2/4} + \frac{\left(\frac{b}{2}\right)^{2n}}{(2n)!} e^{b^2/4} \right\}.$$

It will follow, moreover, from this last expression that the Fourier series (9) converges. From the nature of the function $I(t)$ and consequently also of $I^*(t)$ its Fourier development is unique. It can be shown that $I^*(t)$ satisfies (1) and that equations (6) are satisfied by the corresponding coefficients of (9). In the course of this part of the work, use is made of the results in Watson (l.c.) on page 31, equations (3) and (4).

TRUE GROUND MOTION FROM MECHANICAL SEISMOGRAPH RECORDS

A. B. BRYAN[1]

ABSTRACT

Records of several types of ground motion are obtained with two vertical mechanical seismographs of widely different characteristics, mounted on the same base and recorded on the same moving photographic paper. Companion records obtained are shown in one case to yield substantially the same calculated ground motion although the original records are very dissimilar.

The record obtained with a seismograph is always affected to some extent by the characteristics of the instrument itself and never shows the true motion of the ground upon which the instrument rests. However, if the instrument is a simple mechanical one of known characteristics, the true ground motion can readily be obtained from the original record by a simple but somewhat laborious procedure. For an electrical instrument of the type ordinarily used in geophysical exploration the problem would be much more difficult.

The problem of obtaining true ground motion has been considered in several published papers. H. A. Wilson[2] has discussed the problem thoroughly and has obtained the ground motions corresponding to some mechanical seismograph refraction records. More recently Karl Dyk[3] has described experiments in which records were obtained with a mechanical instrument mounted on a shaking table. The calculated motion of the table, as obtained from the record, agreed well with the directly observed motion of the table. In the present work, records are obtained simultaneously with two mechanical instruments of widely different characteristics mounted on the same base, and it is shown that although the two original records are not at all alike they yield the same calculated ground motion.

The method of calculating true ground motion will be briefly reviewed. The equation of motion of a simple mechanical seismograph is:

$$M(\ddot{x} + \ddot{y}) + R\dot{x} + Sx = 0 \tag{1}$$

where x = displacement of the seismograph mass with respect to the instrument.

[1] The Carter Oil Company, Tulsa, Oklahoma.

[2] H. A. Wilson, Physics, 12, 186, 1932.

[3] Karl Dyk, Bulletin of the Seismological Society of America, 25, 119, 1935.

y = displacement of the instrument and the ground on which it rests.
M = mass of the suspended system.
R = damping constant (viscous damping assumed).
S = spring constant.

From equation (1) it follows that:

$$(2) \quad -\ddot{y} = \ddot{x} + (R/M)\dot{x} + (S/M)x,$$

$$(3) \quad -\dot{y} = \dot{x} + (R/M)x + (S/M)\int_0^t x\,dt + K_1,$$

$$(4) \quad -y = x + (R/M)\int_0^t x\,dt + (S/M)\int_0^t dt\int_0^t x\,dt + K_1 t + K_2,$$

Taking $x=y=\dot{y}=\dot{x}=0$ at $t=0$ as initial conditions, then $K_1=K_2=0$ and

$$(5) \quad -y = x + (R/M)\int_0^t x\,dt + (S/M)\int_0^t dt\int_0^t x\,dt.$$

The instrumental constants (R/M) and (S/M) may readily be evaluated from a record taken with the instrument swinging freely. It may be shown that:

$$(6) \quad (R/M) = (2/T)\log_e r$$

$$(7) \quad (S/M) = (4\pi^2 + \log_e{}^2 r)(1/T)^2$$

where r = ratio of two consecutive swings in the same direction.
T = period of oscillation of the instrument.

In equation 5 the ground displacement (y) is expressed as the sum of three terms. The first term (x) is given directly as a function of (t) by the original record, assuming that the mechanical and optical magnifications are known. The integral in the second term may be evaluated for any value of (t) by graphically computing the area under the (x) curve of the original record from its beginning up to the time (t). A second term may thus be obtained whose ordinate at any time is the area under the (x) curve from the beginning of the record up to that time. Similarly, the integral in the third term may be evaluated for any value of (t) by graphically computing the area under the second term curve from the beginning up to that time, and a third term curve may be plotted whose ordinate at any time is equal to the area under the second term curve from the beginning up

to that time. The ground motion (y) is then obtained by adding together the (x) curve of the original record and the second and third term curves, the latter two being multiplied by the previously determined instrumental constants (R/M) and (S/M) respectively.

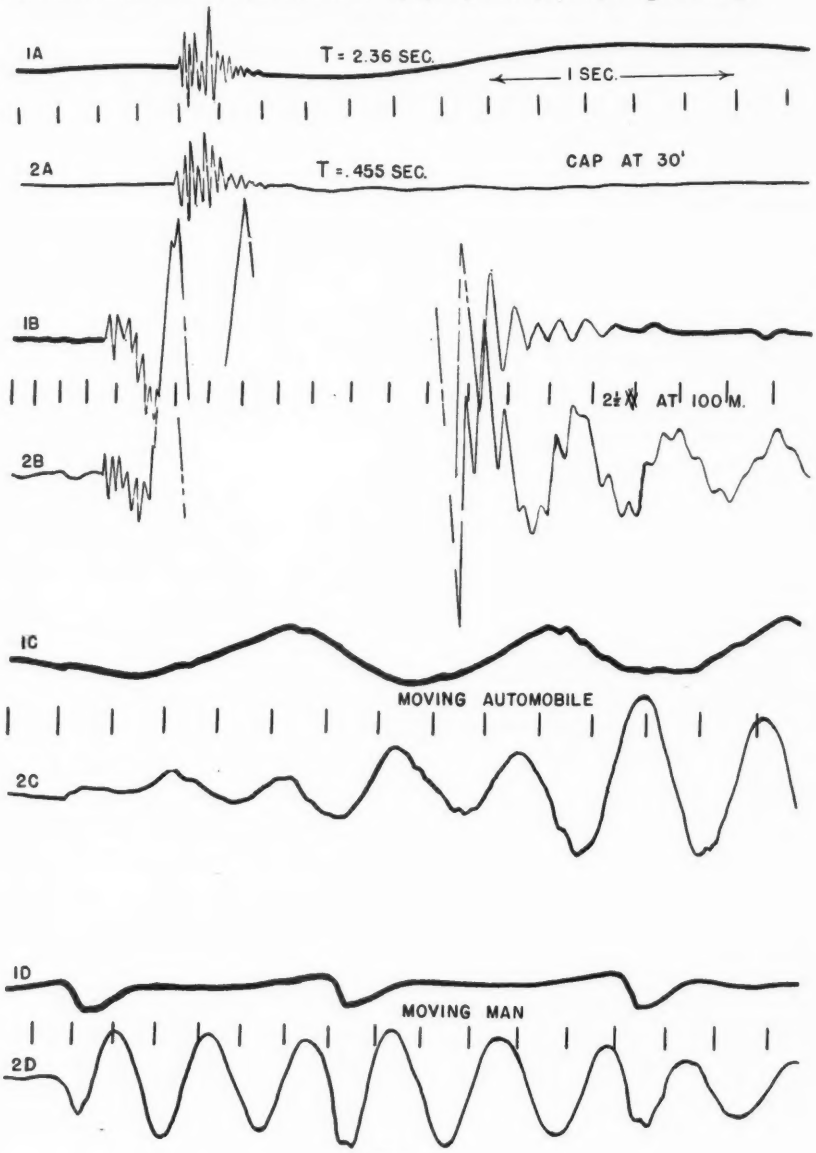


FIG. 1

In Fig. 1 are shown several pairs of records of the same ground motion made with two mechanical instruments of different characteristics. The two instruments were mounted on the same base and recorded on the same strip of photographic paper. The No. 1 instrument had a period of 2.36 seconds while for the No. 2 instrument the period was only .455 second. Both instruments were very little damped, requiring fifteen or twenty seconds to come to rest when

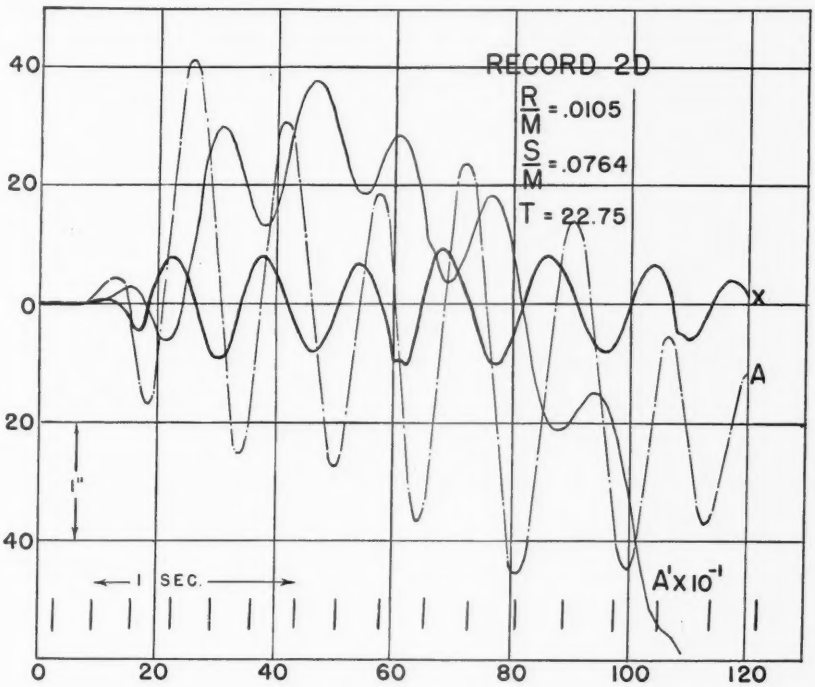


FIG. 2

disturbed. It will be noted that for ground motions obtained with explosives at short distances the records obtained in the two instruments are very much alike although the natural period of the instrument does appear on the latter part of record 2B. For slower motions from other sources the two records are not at all alike.

The original record 2D is shown as curve (x) in Fig. 2. The optical and mechanical magnifications of the instruments were not determined. The ordinates of all curves shown are of course multiplied by this constant. Curve A, showing the area under the (x) curve,

was obtained by tracing the original record on coordinate paper and going through a process equivalent to counting squares under the curve. In this process the horizontal scale must be converted to time units, and variations in paper speed must be taken into account. It was convenient to use one-fiftieth second as the time unit. The period of the instrument was accordingly expressed in fiftieths of a second in all calculations. Curve A' shows the area under the A curve, ob-

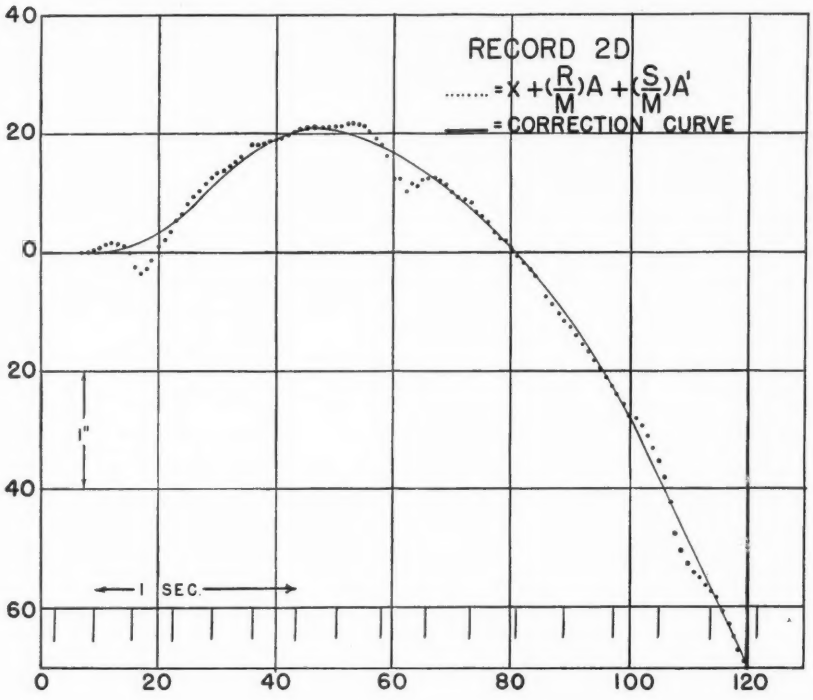


FIG. 3

tained in a similar manner. It will be noted that curves A and A' drift downward and become entirely negative at the end of the record. A drift of this sort is caused by a very slight error in placing the zero axis when the original record is traced or by a slight downward drift of the zero position of the instrument.

The dotted curve in Fig. 3 shows the sum of the (x), A and A' curves of Fig. 2, the latter two being multiplied by the values of R/M and S/M respectively, and should represent $(-y)$ where (y) is the true ground motion. It will be observed that on account of the

previously mentioned drift the general trend of the curve at first rises and then falls rapidly to large negative values. Accordingly, a correction curve, shown as a solid line, is arbitrarily drawn to follow this general trend as closely as possible and is subtracted from the dotted curve to obtain finally the true ground motion, as shown in the upper curve of Fig. 4. The other dotted curve in Fig. 4 shows the same ground motion as calculated in the same way from the com-

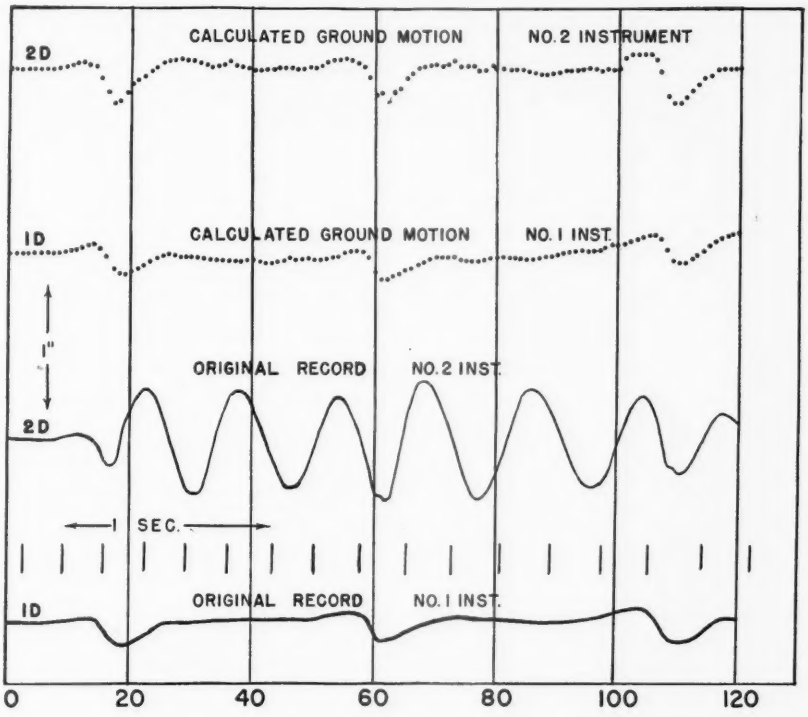


FIG. 4

panion record 1D. In this case no correction curve was used. The constants of this instrument are $R/M = .0116$, $S/M = .00287$ and $T = 118$. It will be noted that except for minor irregularities and differences in amplitude due to different magnification factors, the two instruments yield the same calculated ground motion. The original records are also for convenience reproduced in Fig. 4. It is seen that the No. 1 instrument in this case gives the true ground motion directly and that for this instrument no calculation is necessary.

The principal difficulty with this method of calculating true ground

motion, aside from the labor involved, lies in the necessity for using an arbitrary correction such as that shown in Fig. 3. The assumption is made that very slow motions of the type shown by the correction curve do not exist and hence should be subtracted as a correction. This assumption would for some types of motion be invalid. However, if the motions are known to be reasonably rapid the correction does not introduce any uncertainty. A correction of the sort shown is made necessary by a very slight error in drawing in the zero line on the original record. It may be shown that a correction curve approximately the same as that shown in Fig. 3 would be made necessary if the zero line were drawn 0.1 mm too low at the beginning of the record and 0.6 mm too high at the end, and drafting errors of this magnitude are difficult to avoid.

This work was done at the Rice Institute, Houston, Texas, with instruments loaned by the Humble Oil and Refining Company. The writer is obligated to Mr. N. N. Zirbel for assistance with the experiments and the computations.

AN EXPERIMENTAL STUDY OF THE ELASTIC PROPERTIES OF ROCKS[1]

J. M. IDE[2]

ABSTRACT

Laboratory determinations of Young's modulus, rigidity, and compressibility were made on a set of representative rock samples. These measurements are compared with the theoretical relations between the elastic constants of an isotropic medium. Computed longitudinal and transverse wave velocities are compared with direct field measurements by Leet in granite and norite. Agreement is within 5 per cent for norite, and 20 per cent for granite.

The interpretation of observed elastic wave velocities in the rocks of the earth's crust is our principal basis for inferring its structure at inaccessible depths. The practical geophysicist is interested in time differences for waves travelling along various paths, rather than in the actual velocities in different rocks. However, the actual velocities are of fundamental importance when we deal with broader problems, such as the determination of the types of rock existing at the depths through which earthquake shocks travel. As Jeffreys remarks in his book, "The Earth": "The velocities of elastic waves in the upper layers of the crust give some information about the elastic properties of the materials. If this information is compared with experimental determinations of the properties of the rocks known from geology to be probable, we can proceed a long way towards identifying the main constituents of the crust."

As a basis for reasoning on geophysical problems, it is particularly important to know how well we are justified by experiment in applying to large rock masses the usual assumptions underlying the theory of elasticity.

If we make the customary assumption that a given elastic medium is homogeneous, isotropic, infinitely extended, and that Hooke's law holds, then the following well known relations can be derived:

$$E\beta + 3E/\mu = 9 \tag{1}$$

$$V^2 = \frac{\mu}{\rho}\,\frac{4 - E/\mu}{3 - E/\mu} \tag{2}$$

[1] The research described in this paper was performed at Cruft Laboratory, Harvard University, Cambridge, Massachusetts.

[2] Shell Petroleum Corporation.

$$v^2 = \frac{\mu}{\rho} \tag{3}$$

where E is Young's modulus of elasticity.

β is the cubic compressibility of the medium.

μ is the modulus of rigidity (or shear).

V is the velocity of a longitudinal elastic wave in the medium.

v is the velocity of a transverse wave.

ρ is the density of the medium.

Several other relations can be written in terms of the above quantities and σ, which is Poisson's ratio of lateral contraction to longitudinal extension. Since these formulas are interdependent, the three equations written above will be sufficient for the comparison of theory and experiment.

In this research E and μ were experimentally determined for cylindrical cores of rock, two inches in diameter and ten inches long. The compressibility, β, had been previously measured by Dr. Zisman, of Harvard, on smaller cores of the same rocks. From these data we can test formula (1).

For three rock formations, the Sudbury norite, Rockport granite, and Quincy granite, Dr. L. D. Leet of Harvard University[3] has measured time-distance curves, using equipment similar to that employed in seismic prospecting. From these curves experimental values of V and v were determined. Our measurements on rock cores from these formations enable us to compare Leet's values with wave velocities computed from Young's modulus and the modulus of rigidity, using formulas (2) and (3).

The method of measuring Young's modulus for the rock cores was devised by the author, and is described in detail in the Review of Scientific Instruments, October, 1935.[4] The specimen, with metal foil cemented to its lower end, is placed upright on a thick steel disk. The foil and steel surfaces are separated by a thin sheet of mica. An alternating voltage of variable frequency applied across the electrical condenser thus formed, causes the rock to vibrate by electrostatic traction. When the frequency of the voltage resonates with the natural longitudinal frequency of the specimen, the latter vibrates with increased amplitude, and radiates sound. This is detected by ear or by a piezo-electric crystal receiver. From the fundamental longi-

[3] Physics 4, 375, (1933).

[4] See also J. M. Ide, Proc. Nat. Acad. Sci., February, 1936.

tudinal resonance frequency, f_0, Young's modulus is computed from the formula:

$$f_0 = \frac{1}{2l}\sqrt{\frac{E}{\rho}}$$

where l is the length of the specimen, and ρ its density. The method is simple and rapid, and accurate within a few tenths of one per cent.

Since the modulus of rigidity is related to the natural frequency of torsional vibration by a formula exactly analogous to this, it can be measured by an arrangement for setting the rock cylinders into torsional vibration. The method of this research is to fasten a small steel strip rigidly in a saw cut at one end of the rock sample, and produce torsion of the strip by two small electromagnets carrying alternating current from a variable frequency oscillator. The electromagnets are set on opposite sides of the strip, and produce a torque on it which is transmitted to the end of the rock. A similar device at the opposite end of the sample, with the electromagnets connected to an amplifier, is used to detect the vibrations of the cylinder. This arrangement is a modification of one used by Wegel and Walther of the Bell Laboratories,[5] in a study of the vibrations of metal rods.

The compressibilities of these rocks were measured and the measurements described in detail by W. A. Zisman,[6] using a hydraulic testing machine to compress the specimen, while the yielding of the rock was measured by means of a sensitive rocking mirror piezometer. Petrographic descriptions of the rock samples may be found in Zisman's papers.

The results of our measurements, as applied to formula (1), are summarized in Table I. The table gives average values for three or more samples of each type of rock. The percentage deviation from the formula is given to the nearest per cent.

The table shows that the theoretical formula which connects E, β and μ fits the experimental data with varying degrees of accuracy. For copper and steel, and for the diabase,—a compact fine-grained igneous rock, there is agreement between theory and experiment to about one per cent. This is excellent for measurements of elastic properties. For rocks which are less compact than the diabase, more porous, or composed of larger crystals, the formula does not hold so

[5] Wegel and Walther, Physics, 6, 141, (1935).
[6] Zisman, Proc. Nat. Acad. Sci. 19, 653 (1933); 19, 666 (1933).

well. The norite, quartzitic sandstone, marble, and granite show disagreement increasing from three per cent to nineteen per cent. The Quincy granite is very coarse-grained, and the random variations in elastic properties from one sample to another are relatively large (fifteen per cent).

TABLE I

EXPERIMENTAL TEST OF FORMULA $E\beta+3\frac{E}{\mu}=9$

Substance	$E\beta+3\frac{E}{\mu}$	*Deviation from 9*
Steel	8.88	− 1
Copper	9.13	+ 1%
Vinal Haven Diabase	9.12	+ 1%
Sudbury Norite	8.76	− 3%
Quartzitic Sandstone	8.15	− 9%
Marble	7.48	−17%
Rockport Granite	7.88	−12%
Quincy Granite	7.31	−19%

Another conclusion which follows from the data is that the disagreement between theory and experiment is always in the same direction, that is: the experimental value of $E\beta+3E/\mu$ is uniformly smaller than we should expect. The quantity $E\beta$ is in general less than twenty per cent of the sum of the two terms of the formula, so that the quantity $3E/\mu$ is the one which shows the largest disagreement with the theory.

We conclude then, that formula (1) agrees with experiment for metals and for a compact diabase rock, but that experimental values of $E\beta+3E/\mu$ may be from nine to nineteen per cent lower than the formula predicts for such rocks as marble, quartzitic sandstone, and granite. We can also make a reasonable guess that disagreement between theory and experiment will be still larger for the less compact sedimentary rocks such as shales, conglomerates and soft sandstones.

In order to test formulas (2) and (3) by experiment, we compute longitudinal and transverse wave velocities from our measured values of elasticity and rigidity. We then compare these computed wave velocities with those derived by Leet[7] from time-distance curves taken in the field. This comparison is made in Table II.

It is evident from the table that for the Sudbury norite the formulas

[7] *Op. cit.*

agree with experiment to within about five per cent, while for the granites the figure must be raised to nineteen per cent. This result checks closely the comparison in Table I, where the agreement for norite and granite is three per cent and nineteen per cent respectively. Thus each of the three formulas from elastic theory is checked by experiment to about the same degree of accuracy. This accuracy is high for diabase and norite, and relatively low for granite.

TABLE II

COMPARISON OF FIELD AND LABORATORY DETERMINATIONS OF ELASTIC WAVE VELOCITIES IN ROCKS

Rock	*Transverse Wave Velocity v km./sec.*	*Longitudinal Wave Velocity v km./sec.*	*Differences*	
			Trans.	*Longit.*
Sudbury Norite	3.57 Lab. 3.49 Field	5.93 Lab. 6.22 Field	− 2%	+ 5%
Rockport Granite	2.66 Lab. 2.70 Field	4.18 Lab. 5.14 Field	+ 1%	+19%
Quincy Granite	2.93 Lab. 2.48 Field	4.28 Lab. 4.96 Field	−18%	+14%

The reliability of the laboratory methods employed in this research is established by the excellent agreement with theory obtained for steel, copper, and diabase. Thus we must conclude that the discrepancies found for other materials are real, and that such rocks are therefore imperfect elastic media. The formulas of elastic theory are derived on the condition that the medium is homogeneous, and possesses the same properties in all directions. It is certainly reasonable that some types of rock should fit these conditions better than others. Our experiments show this to be true, and give the sign and order of magnitude of the discrepancies to be expected.

It is possible that if the elastic constants of rock samples were measured at the pressures which exist even at a depth of a few hundred feet in the ground, theory and experiment might agree more closely. It is possible also that dispersion, or a variation of velocity with frequency, exists in rocks to a sufficient degree to explain part of the discrepancy between laboratory and field measurements. The laboratory experiments on elasticity and rigidity were carried out at frequencies near 10,000 cycles per second, while Dr. Leet measured the group velocities of transient shocks.

These questions are vital to the solution of important problems in geophysics, but they can be answered only by further experimental work on the elastic properties of rock, under the conditions which exist at great depths in the earth.

In conclusion, the experiments described in this report suggest caution in applying the theory of elasticity to the surface rocks of the earth's crust, whether they be sedimentary or granitic in character. On the other hand, the results for diabase rock show that the theory can be expected to hold rigorously for the fine-grained igneous rocks which are presumed to lie beneath the oceans and the continental granites.

INTERSTITIAL WATER SATURATION IN THE PORE SPACE OF OIL RESERVOIRS[1]

JAMES A. LEWIS[2] AND WILLIAM L. HORNER[2]

ABSTRACT

The presence of water in oil reservoir cores has been observed for several years by analysts using a direct method for fluid content determinations. The actual amount of water that is present in the sand is of importance in the estimation of oil reserves as well as in determining the type of fluid which a particular stratum will produce under conditions favorable for flow into the well. The core analysis profile of a well cored with oil (using Baker Core Barrel) is compared with three other wells which were cored with water. The analysis of the former core is in close agreement with respect to oil and water saturations, 30.58%, 44.40% respectively, with other cores in the area, and points to the conclusion that water was distributed throughout the sand section with no indication of gravitational separation at the time of coring. Coring with oil or the addition of indicators to the fluid used during coring and the subsequent analysis of same are methods for determining correct interstitial water saturation.

INTRODUCTION

The presence of water in oil reservoir strata has been observed for the past several years by analysts using a direct method of analysis to determine the fluid saturation of oil well cores. The origin of this water saturation has been the subject of many discussions, and at present sufficient information is at hand to show that the data of per cent water saturation should be given weighty consideration when determining whether a well will produce water or commercial oil. It will be shown in the discussion which follows that water is present in the interstices of some oil sands at the time of coring and, depending on the actual amount of water present, is produced simultaneously with the oil or is held in the pore spaces throughout and after the time of commercial operation. Water content data also are necessary in making quantitative estimates of the space available in pools for the accumulation and storage of oil. Calculations of fluid flow and estimates of per cent recovery and original oil content of reservoirs have long been made by assuming the complete occupancy of the pores by hydrocarbons. This assumption seems logical, for it is often found that the oil strata yield pipe line oil without noticeable amounts of free water. However, the presence of water in the pores materially reduces the space available for the movement of and occupancy by oil, and consequently this paper may be of in-

[1] Read before the Society of Petroleum Geophysicists at Tulsa, March 20, 1936.

[2] Core Laboratories, Inc., Dallas, Texas

terest to investigators of methods for predicting recovery and subsurface fluid flow. It is interesting also to speculate as to the effect of this interstitial water saturation on the electrical methods in use at the present time in locating oil bearing horizons. The relative uniform distribution of this water content throughout a sand of given equal characteristics, as pointed out later, as well as the fact that this water is in the position of contact in films of variable thickness with the sand, suggests to us the possibility of relatively low resistivities. In other words, it should be significant that the pores of many oil pools, instead of being filled entirely with oil, are filled 10 to 50 per cent of their volumes with electrolytic solutions. Absolute resistivity values for oil sands in situ should therefore be lower than values for dry or artificially oil-saturated rocks. The presence of this water saturation in oil reservoirs is of importance in the evaluation of properties for secondary recovery methods as well as for those under natural production, and should be given consideration in the completion of every well. The presence of interstitial water in an oil-bearing formation introduces the proposition that a definite oil-water contact does not exist in most reservoirs, and that water production in a well is not necessarily derived from a definite water strata or structural position, but can be produced from the same strata as is the oil. Geologic time has permitted all of the oil to rise above the water table in most homogenous sands, but has not let all of the water drop out of the oil zones. The contact usually referred to is, therefore, not a plane where one hundred per cent oil saturation overlies one hundred per cent water saturation, but is a plane below which no oil production is obtained.

ORIGIN AND SIGNIFICANCE OF WATER SATURATION

The origin of this interstitial water in oil producing strata is open to question. However, it seems reasonable to us that, where the water percentage is not excessively high for sands of given characteristics, it is either connate water or water derived from the connate water which was present in reservoirs before the accumulation of oil, and because of the preferential wetting of silica by water in comparison to oil, is held in smaller capillaries and in openings where a condition of equilibrium exists between the quantities of water present and the diameter of the capillaries. As pointed out by Garrison (1) "In order that a well shall produce nothing but oil, it is necessary that the oil flow to the well bore through the larger pores at such a rate that it

does not cause the water to advance from its equilibrium position in the smaller capillaries to a point where it is forced along with the oil."

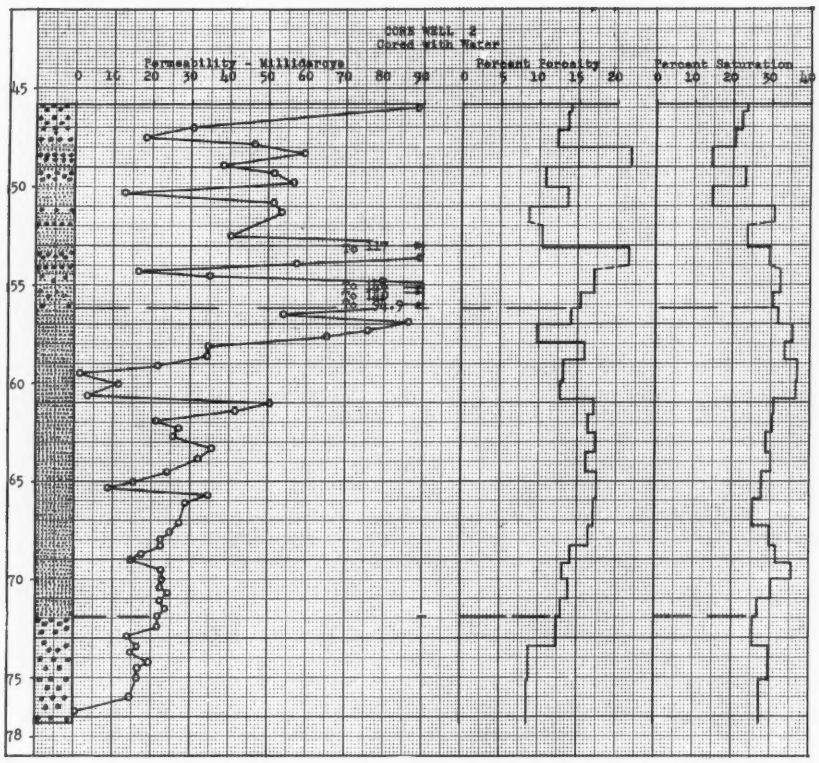
This condition of equilibrium represents varying percentages of water saturation in sands of different physical characteristics. Those sands which are formed of small grains will contain a higher percentage of water in the state of equilibrium than will those with larger average grain size. Likewise, the displacement of water by oil will be more effective during the replacement period in the more permeable formations, and will result in higher oil contents being found in these strata. The physical characteristics of a sandstone may be determined by the use of apparatus (2) (3) such as is found in petroleum production and research laboratories at the present time, and these results, when combined with determinations of fluid contents, yield data which indicate that there is a critical water saturation value for each and every sand that is worthy of note.

DIRECT METHOD TO DETERMINE WATER SATURATION

The presence of water in oil sand cores has been observed by the authors in each of several hundred determinations, and attention has been directed to investigating the question of its source both by direct and indirect methods. As is customary practice in the Appalachian Fields, practically all cores have been taken with a water head of from 60 to 80 feet in the hole. (Baker Method.) This practice has led to the general belief that the water which was found in all saturation samples was the result of penetration of coring fluid, and that water did not occur in a natural condition within the pores of the sand. Subsequent investigation and a change in procedure, whereby oil was used during the coring operation instead of water, facilitated the direct determination of water content, and at the same time presented an opportunity to compare the oil absorption and per cent oil saturation with previous results on cores taken with water.

The first four core analysis profiles included herewith were drawn from laboratory results of cores from four wells located at the corners of a rectangle approximately 380 feet by 480 feet on the sides. C.W. 2 and C.W. 3 as well as C.W. 4 and C.W. 5 are the pairs of wells which are spaced 380 feet with C.W. 2 and C.W. 5 on one 480 foot side, and C.W. 3 and C.W. 4 on the other 480 foot side. The order of coring was that designated by the numbers for the wells, with C.W. 2, C.W. 3 and C.W. 5 being cored with water, and C.W. 4 being cored with oil. The area in which these cores were taken shows very

little structural relief on top of the sand, and within the distances represented by these wells, is of only general significance. A line across the diagonal of the rectangle from C.W. 4 to C.W. 2 follows a line from the structural low to the structural high. The inconsistency in total sand thickness is the result of an irregular base for the bottom of the sand. The natural reservoir pressure in the sand does not exceed



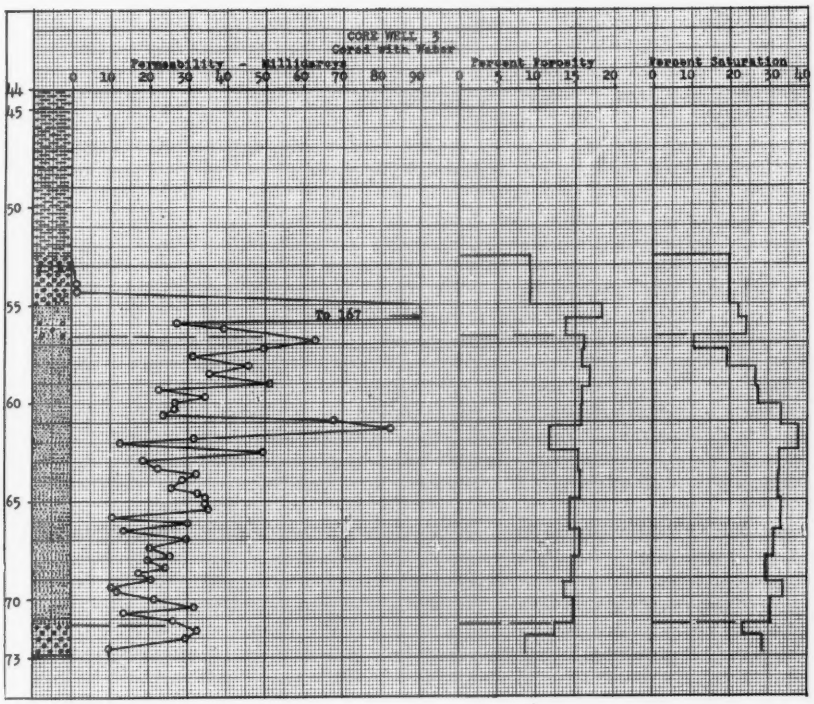
Depth, Feet	Feet Sand	Avg. Permeability, Millidarcys
45.8–56.2	10.0	63.4
56.2–71.9	15.7	28.8
71.9–77.3	5.4	14.9

FIG. 1. Core Analysis Profile C.W. 2. Cored with Water.

15 lbs. per square inch gauge. The oil and water content of the sand was determined by distillation of fluids from the consolidated sample, and is the method which is in general use in the Appalachian Field for the analysis of oil field cores. By this method of analysis, the oil

and water content of a sample are determined simultaneously, and the results have been proven to such an extent that the proper evaluation of a property for expected recovery is a matter of routine analysis.

Reference to Figs. 1, 2, 3, and 4 show permeability, porosity and oil saturation figures which are characteristic of this area. With the

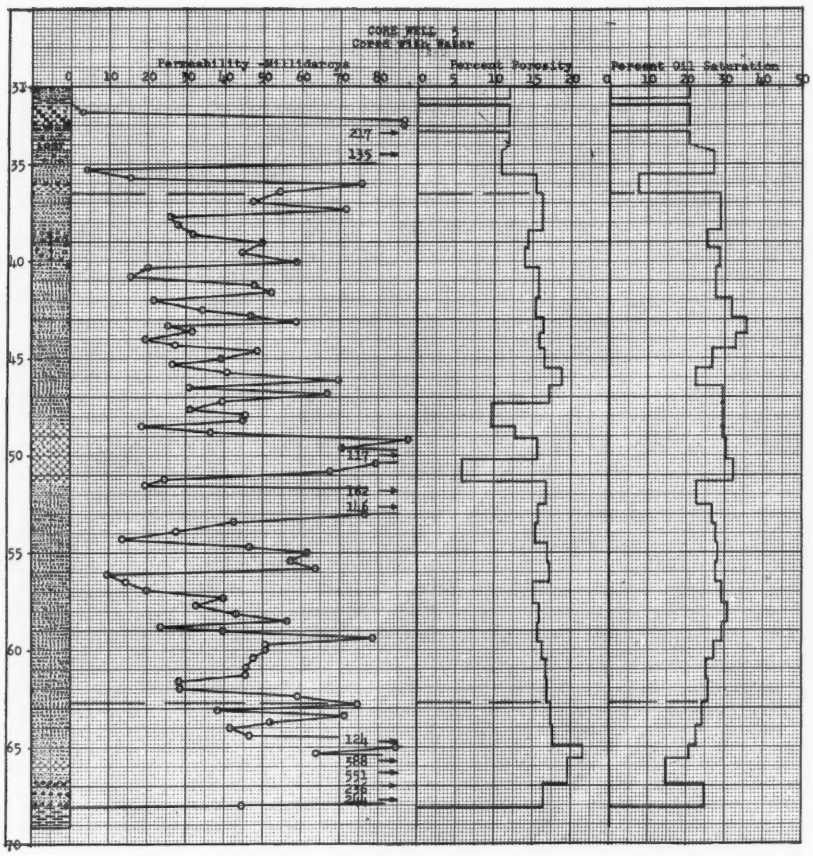


Depth, Feet	Feet Sand	Avg. Permeability, Millidarcys
52.5–56.6	3.9	38.4
56.6–71.3	14.7	31.0
71.3–72.8	1.5	22.2

FIG. 2. Core Analysis Profile C.W. 3. Cored with Water.

exception of certain small thicknesses of sand which frequently are of a conglomeratic nature, the distribution of permeability values is such as to show a uniform sand with uniform flow characteristics. The results for these wells are summarized in Table I. The core in each instance has been divided into three sections on the basis of sand structure; the upper and lower divisions including in most cases

the conglomeratic strata which are considered too irregular for profitable operation. The middle division, section #2, includes the sections of fine-grained sand which contain the higher percentage oil saturation values, and in turn, the highest oil contents. The bottom section

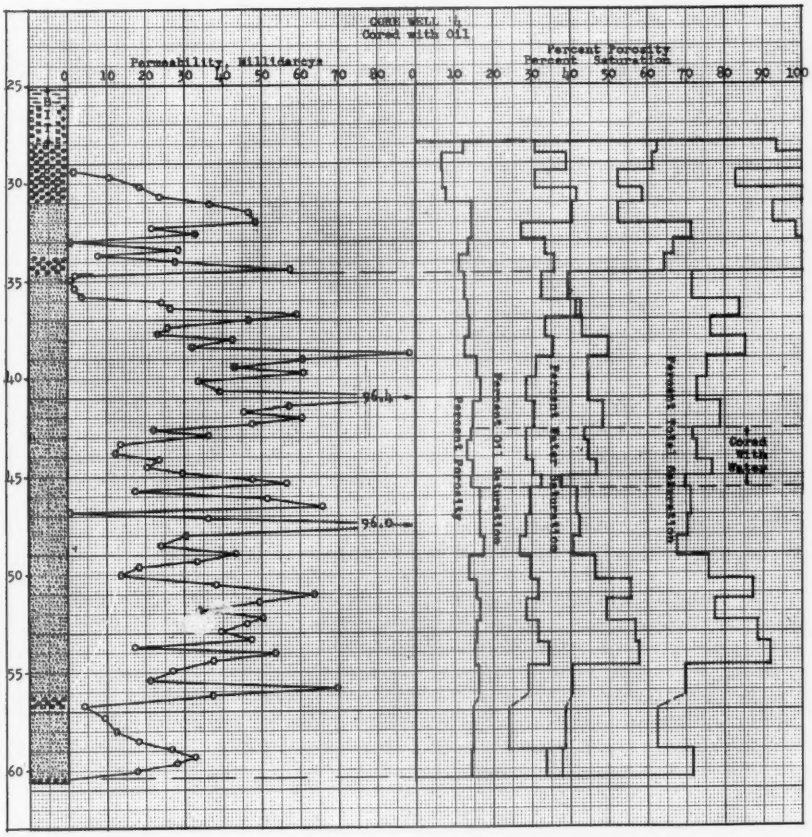


Depth, Feet	Feet Sand	Avg. Permeability, Millidarcys
31.0–36.5	4.9	77.9
36.5–62.7	26.2	49.0
62.7–68.1	5.4	217.0

FIG. 3. Core Analysis Profile C.W. 5. Cored with Water.

of sand in C.W. 5, Fig. 3, possesses high permeability, with low oil saturation and high water saturation, and is a section which would act as a good path of travel for fluid entering or leaving the hole

should it be left open in the completed well. In this case, under equal pressure differentials, the bottom 4.5 feet of formation has a productive capacity equal to the middle 26.2 feet of sand, and under natural production, under equal pressures, would produce a quantity of



Depth, Feet	Feet Sand	Avg. Permeability, Millidarcys
26.0–34.6	8.6	16.8
34.6–60.4	25.1	35.6

FIG. 4. Core Analysis Profile C.W. 4. Cored With Oil.

water equal to or greater than the amount of oil produced from the middle division. It is obvious that it is important to know the sections of sand in a well that are producing a particular type of fluid in order that the proper completion of a well for the exclusion of water

may be known. It is likewise necessary and valuable to determine the net sand thickness which may produce oil under a given pressure at a definite rate, in order more thoroughly to evaluate the real worth of a productive zone.

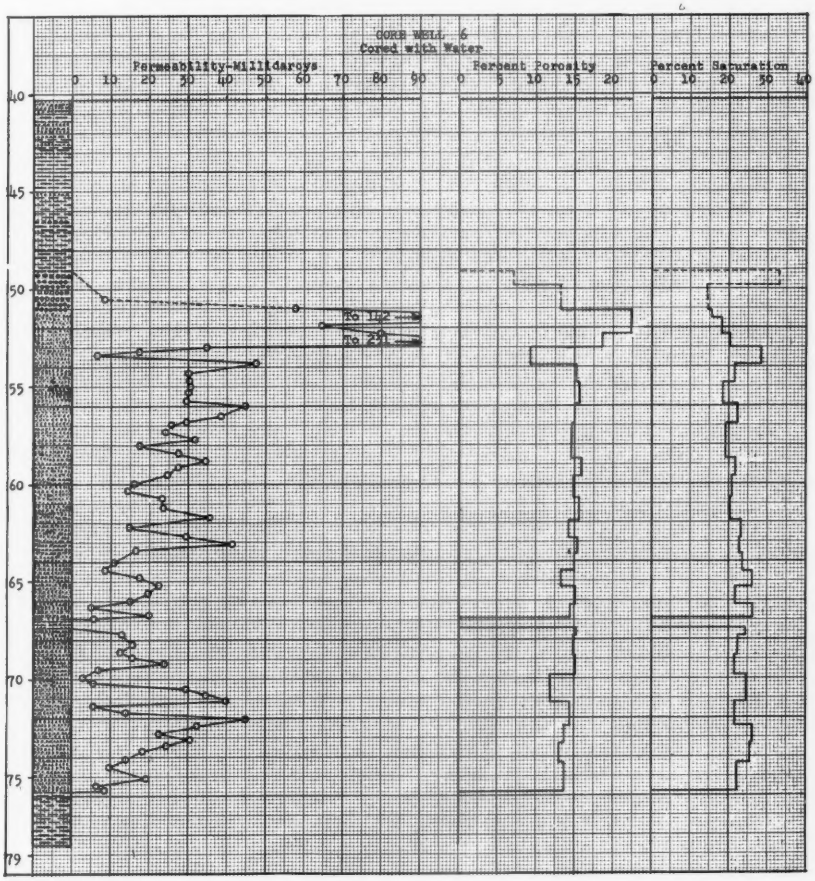
The core analysis profiles of the three wells just referred to were obtained by using water as the coring fluid. C.W. 4, Fig. 4, in the immediate vicinity of the previous wells was cored with oil from the top of the sand to a point 16.5 feet in, where water was used to core one screw. During the time that the well was cored with oil, the hole was bailed down after each three foot screw, and there was no evidence of water being present in the cavity. After taking the one screw with water in the hole, all fluids were then removed, and the coring continued with oil. The detailed laboratory results in Fig. 4 represent permeability determinations at the points of the circles with fluid content determinations represented by solid lines for approximately each foot of sand. All values are direct, and the actual values are shown on the upper line.

The permeability values for this well are in agreement with the specific and average values of the previous wells and the distribution is normal. It is apparent that even though this well was cored with oil, the oil saturation of the core was in close agreement with the previous wells. Also it is apparent that the per cent water saturation is distributed from top to bottom of the core in all sections of the oil zone and gives no evidence of gravitational separation. In addition to the above similarities between this core and the ones previously mentioned, another point is the per cent total fluid saturation in this sand, which is comparable to that present in the former cores. With a condition of 75% total fluid saturation with the water, oil and gas present in a state of equilibrium, and with an equal pressure around the sand biscuit at the time of coring, a condition suitable for fluid absorption where the Baker Method is being used is far beyond reasonable expectation.

Inspection of the summary sheet gives the calculated average saturations for these four cores within the limits 28.07 to 30.65% oil; 39.56 to 49.05% water, and 67.63 to 79.04% total fluid saturation. In particular, the well which was cored with oil had an oil saturation of 30.58% (not the upper limit of oil saturation values for these cores), and a water content of 44.40% saturation, which also is between the limits of maximum variation.

Fig. 5 is the core analysis profile of a well where the water satura-

tion is higher than the amount found in any of the previous wells. With an average water saturation of 51.52%, and the low oil saturation of 21.97%, it is significant that water was the fluid that was



Depth, Feet	Feet Sand	Avg. Permeability, Millidarcys
49.1–51.1	2.0	23.1
51.1–75.8	24.2	26.3

FIG. 5. Core Analysis Profile C.W. 6. Cored with Water.

bailed from this well. Under long continued operation, this well would not be a commercial producer, and its completion is not justified.

TABLE I

CORE WELLS #2, #3, #4, #5, #6

SUMMARY

Well No.	Net Sand Thickness Feet	Per cent Porosity (1)	Per cent Oil Saturation (2)	Per cent Water Saturation	Per cent Total Saturation	Per cent Unfilled Voids	Bbl. Oil per Acre Foot	Bbl. Oil per Acre	Average Permeability, Md.
Section 1									
C.W. 2	10.0	14.60	24.01	54.35	78.36	21.64	272	2,720	63.4
C.W. 3	3.9	11.72	22.65	63.61	86.26	13.74	206	803	38.4
C.W. 4*	8.6	11.08	33.85	62.70	96.55	3.45	291	2,503	16.8
C.W. 5	4.9	12.36	18.87	60.46	79.33	20.67	181	887	77.9
C.W. 6	2.0	11.04	18.80	55.29	74.09	25.91	161	322	23.1
Section 2									
C.W. 2	15.7	15.01	30.65	42.83	73.48	26.52	357	5,605	28.8
C.W. 3	14.7	15.00	29.99	49.05	79.04	20.96	349	5,130	31.0
C.W. 4*	25.1	14.92	30.58	44.40	74.98	25.02	354	8,885	35.6
C.W. 5	26.2	15.29	28.07	39.56	67.63	32.37	333	8,725	49.0
C.W. 6	24.2	14.84	21.97	51.52	73.49	26.51	253	6,123	26.3
Section 3									
C.W. 2	5.4(3)	—	—	—	—	—	—	—	14.9
C.W. 3	1.5(3)	10.10	25.65	43.05	68.70	31.30	201	302	22.2
C.W. 4*	(4)	—	—	—	—	—	—	—	—
C.W. 5	5.4	18.26	21.03	51.77	72.80	27.20	298	1,609	217
C.W. 6	(5)								

* Cored with oil. All others with water.
(1) Gas expansion.
(2) Retort.
(3) Only part of section cored.
(4) Not penetrated.
(5) Absent.

OIL AND WATER SATURATION VS. PRODUCTION

Sands of different physical characteristics have critical water saturation values that are controlled by the texture of the sand, and whenever the actual value exceeds this limit, water production can be expected; and whether or not oil is produced depends on the relative amounts of both fluids that are present and their relationship to the total pore space.

The results which follow are for cores in the Bradford, Pennsylvania field on which production records are available. The exact water production in each case has not been recorded but is noted when the amount becomes greater than 3% of the total fluid pumped (Table II).

TABLE II

FLUID SATURATION AND PRODUCTION FOR BRADFORD, PA., CORES

Core No.	*% Water Saturation*	*% Oil Saturation*	*Water Production**
1	22	49	No
2	28	39	No
3	28	46	No
4	29	48	No
5	30	45	No
6	33	56	No
7	34	50	No
8	34	41	No
9	34	29	No
10	35	35	No
11	36	38	No
12	36	30	No
13	39	48	No
14	39	40	No
15	39	32	No
16	40	45	Yes
17	40	35	No
18	40	24	Yes
19	41	38	Yes
20	42	42	Yes
21	43	42	No
22	43	39	No
23	43	17	Yes
24	45	41	Yes
25	46	48	Yes
26	46	26	Yes
27	51	38	Yes
28	57	28	Yes
29	59	27	Yes

* Noted when water production exceeded 3% of the total fluid production during the first week on pump.

INDIRECT METHOD OF WATER DETERMINATION

Although oil may be used during coring to secure satisfactory samples for interstitial water determinations, the method has cer-

tain inherent difficulties which can be eliminated by the method of adding certain chemicals to the drilling fluid and the subsequent analysis of the water from the sand samples. This procedure (4) makes possible the division of the water content into that which was natural and that which was derived at the time of coring. Tests of this nature on Bradford Sand have shown that the amount of penetration is very small, and that for all practical purposes is of little value. As more permeable sands are cored which in their natural condition possess a lower total fluid saturation than does Bradford Sand, this question of contamination should be investigated more thoroughly. The character of the mud, the pressure differential which exists between the mud and the sand, as well as the contact area and the permeability, are factors which control the amount of penetration of the drilling fluid, and each should be given due consideration before a correct interpretation can be applied to interstitial water saturation.

CONCLUSIONS

From these data the following statements are given in concluding:

1. Natural interstitial water of varying degree of saturation has been found to exist in certain oil reservoirs at the time of coring. The degree of saturation depends on the character of the sample as well as the distance the well is located from the advancing front of the edge-water encroachment.

2. The interstitial water is distributed throughout the complete sand section, and gives no indication of gravitational separation.

3. These data indicate that a definite oil-water contact does not exist in all reservoirs, and that water production in a well is not necessarily derived from a definite water strata or structural position, but can be produced from the same strata as is the oil.

4. In addition to the usual factors in calculating productivity, the specific fluid saturation values, as well as the ratio of water and oil, are the controlling factors in determining the commercial value of a well.

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SOME ASPECTS OF MULTIPLE RECORDING IN SEISMIC PROSPECTING

PAUL W. KLIPSCH*

ABSTRACT

Considerable attention has been given to the use of more than one geophone on each recording channel with the hope of increasing the signal-to-noise ratio, where "signal" is taken to mean "recorded reflection" and "noise" means any undesired recorded amplitude. The average expectancy of this gain is evaluated analytically, and the statistical distribution examined. Since at different instants this gain possesses different values dependent on a large number of causes, these causes are assumed to be random in nature; a justification of this and other assumptions is explained, and the distribution of gain in terms of frequency of occurrence is expressed in the form of a probability curve. From this analysis an answer can be inferred to the question, "if n is the number of geophones, what is the probability that a gain of $k \cdot \sqrt{n}$ will occur?" The average expectancy of this gain is shown to be

$$\overline{G} = \frac{2}{\sqrt{\pi}} \cdot \sqrt{n}.$$

This leads to the conclusion that a large number of geophones per channel is necessary to obtain consistently worthwhile values of gain in signal-to-noise ratio.

Aspects incident to application are discussed, and sources of noise which are not random are mentioned.

INTRODUCTION

Multiple recording, or the use of more than one geophone per recording channel, has for its object an increase in the ratio of reflected energy to other disturbances. Refracted energy, wind noise, seismic unrest, and uncontroled vibrations from traffic or other causes all tend to mask the ground motion which results from reflection from a deeply buried layer.

The subject of multiple recording has been treated by Born[1] using Fourier analysis to show that certain components of the ground motion could be eliminated from the recording by using destructive interference, much in the manner of directive arrays of radio antennae.

The recording may be regarded as composed of several components; the signal or reflection, regular noise, and irregular noise. The regular noise is that resulting directly from the shot; irregular noise includes seismic unrest and wind noise. The proposed method had for its purpose the elimination of certain components of the regular noise.

There is considerable justification for an assumption that all noise

* Independent Exploration Company.

[1] Born, W. T., "Notes on the Use of Multiple Shot Holes and Multiple Geophones," presented at the Dallas convention, Soc. Petroleum Geophysicists, November 29, 1935.

be treated as irregular or random in its nature. If this be done, it is necessary to attack the problem from a different viewpoint involving the statistical approach.[2] It is the purpose of this paper to justify this basis for the study of the problem, and to carry out an analysis based on probability.

In order to develop the idea that the "ground roll" or refracted energy is random in its nature, it is necessary to review some of the fundamentals involved. This leads naturally to a study of the nature of the original disturbance, the paths through which the disturbance is propagated, and the manner in which superposition takes place.

THE NATURE OF AN ARTIFICIAL SEISMIC DISTURBANCE

The impulse generated by an explosion may be considered as a damped wave train. Considerable dispute arises as to the exact form of the wave sent out, but it should be safe to assume that the impulse gives rise to a ground motion which consists of a band or spectrum of frequencies, each (frequency) component of which consists of a damped wave train. Such a component can conveniently be represented by a curve such as Fig. 1. Some components are probably

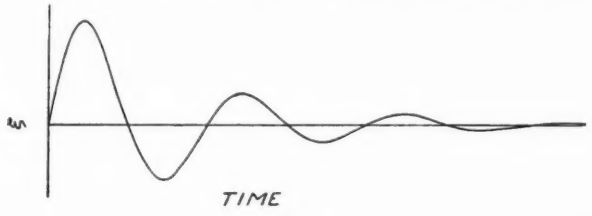

FIG. 1

much more highly damped than others; it would be expected that all such components differ in amplitude, frequency, and damping, and suffer different degrees of attenuation in transmission.

Fig. 1 can also be taken to represent an amplitude-space curve of a particular component of the traveling wave; in fact it is the traveling wave in space which, passing a given point, gives rise to the amplitude-time curve appropriate to that point.

SUPERPOSITION RESULTING FROM MULTIPLE PATHS

The various ways by which elastic waves can travel from the shot point to the recorder provide an almost infinite number of motions

[2] Klipsch, P. W., a prepared discussion following Mr. Born's paper. See footnote 1.

at the recorder. In Fig. 2, a three layer earth is represented, in which the longitudinal velocities are indicated. In addition to the longitudinal velocites, there are the transverse and rotational velocities.

Evidently, the impulse or wave train generated by the explosion will be propagated over various paths at a large number of velocities, arriving at the recording station as a series of superimposed wave trains, displaced in time, distorted in frequency characteristics, and altered as to magnitudes. The summation of these wave trains gives rise to a very complicated motion of the ground at any recorder, the complexity rendering an analytical attack practically impossible. A practical attack still remains, however, in a treatment of the complicated motion as random in nature.

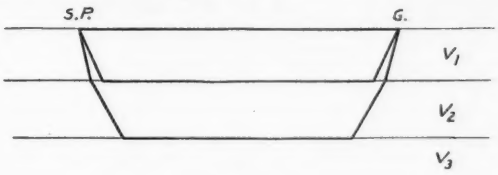

FIG. 2

Almost any event can be regarded as having a cause. For example, the flip of a coin shows heads. This is a result of its previous history, and the number of times it turned over. The causes are regarded as undetermined. In spite of the existence of causes, undeterminable in extent, the coin "obeys" the laws of probability. Similarly, in the case of a seismic refraction shot, the ground motion can not be determined *a priori;* the causes are undetermined until after the shot. Experience shows that no two records are alike, and even after the shot has been fired and the record analysed, it is impossible to learn all there is to know about the causal differences between any two locations. Enough independent causes, occurring at random, exist to justify the statistical approach. If an event, in the form of a particular ground motion, takes place at a point at a certain time, it will occur at a nearby point at some other time, the difference being expressible in terms of a phase angle of some particular frequency component. Even if the distance to the nearby point is not chosen at random, all phase angles within a limited range can be deemed equally likely. Several equidistant colinear points will experience the event at different times, and, excepting first breaks or other regularly identifiable events, the phase angles of the event will be generally random in nature. All values of phase angles are thus equally likely.

One more important principle must be offered before the problem can be stated in a form capable of solution. This is that each frequency component, of which the disturbance is composed, can be treated separately, that all other frequency components obey similar laws, and that the total resultant is the summation of the separate components.

GAIN IN SIGNAL-TO-NOISE RATIO

In applying the principles of multiple recording, it is assumed that an array of shot holes or recorders can be sufficiently scattered so that the noise componenets add up out of phase, and at the same time that the array is sufficiently compact so that the signal, or reflection, adds up in phase. That this is possible depends on the relative apparent surface velocities of the signal and various components of the noise.

Finding the signal-to-noise ratio gain thus depends on first finding how much noise will be reproduced by the array. If n is the number of geophones in the array, the signal amplitude will be n times as great as for a single geophone. The noise, presumably approximately of equal amplitude at each geophone, but having different time or phase displacements, should produce a resultant less than n times the amplitude for a single geophone. The gain in signal-to-noise ratio would then be the ratio of gain in signal amplitude to gain in noise amplitude. The problem is then one of finding the noise amplitude as a function of n.

ANALYSIS

It was shown by Rayleigh[3] that the resultant of a large number of isoperiodic vibrations of arbitrary phase does not converge on any definite value. The average intensity is equal to n, where n is the number of sources of vibrations of unit amplitude, but for more than a superficial study it is necessary to inquire into the relative probabilities of resultants lying within certain assigned limits. It should be possible to state the probability that the resultant will be less than some specific value, say some function of n.

The value of the probability arrived at by Rayleigh is

$$P = 1 - \epsilon^{-r^2/n} \tag{1}$$

[3] Rayleigh, "On the Resultant of a Large Number of Vibrations of the Same Pitch and of Arbitrary Phase" Philos. Mag. X, 1880, page 73 (Scientific Papers, Vol. 1). See also "Theory of Sound" Vol. 1, pages 35–42.

where

P = probability of a resultant amplitude less than r,
r = resultant amplitude, r^2 = resultant intensity,
n = number of unit vibrations of the same pitch and arbitrary phase.

Equation (1) is plotted in Fig. 3, curve for $n=\infty$, giving P as a function of r^2/n.

AVERAGE VALUE OF RESULTANT

The average value of the resultant may best be regarded as the *expectation* which is defined by Coolidge[4]

$$\sum_{i=1}^{i=n} p_i s_i \tag{2}$$

where p_i is the probability of occurrence of an amount s_i. Thus the average value

$$\bar{r} = \sum r \Delta P.$$

Using the method of the Calculus,

$$\bar{r} = \int_0^1 r \cdot dP \tag{3}$$

$$= \int_0^\infty \epsilon^{-r^2/n} \frac{2r}{n} r \, dr$$

$$= \sqrt{n} \int_0^\infty \epsilon^{-r^2/n} \left(\frac{r^2}{n}\right)^{1/2} d\left(\frac{r^2}{n}\right)$$

$$= \sqrt{n}\Gamma(3/2)$$

$$= \sqrt{n} \cdot \frac{\sqrt{\pi}}{2}. \tag{4}$$

It has already been shown[3] that the average intensity

$$\bar{r}^2 = n. \tag{5}$$

This may easily be verified by integrating the equation

$$\bar{r}^2 = \int_0^1 r^2 \cdot dP \tag{6}$$

[4] Coolidge, Julian Lowell, "An Introduction to Mathematical Probability," Oxford, 1925.

$$= \int_0^\infty \epsilon^{-r^2/n} \frac{2r}{n} r^2 dr$$

$$= n. \tag{7}$$

PROBABILITY OF A RESULTANT GREATER THAN AVERAGE

To illustrate the practical implications of the statistical distribution, consider the problem, "what is the probability of obtaining a resultant greater than the average?" The average resultant is given by (4) and we wish to find the probability that r exceed $\bar{r}$. The probability, P, that the resultant is less than r, is given graphically in Fig. 3 as a function of r^2/n. Putting $r=\bar{r}$ gives $(\bar{r})^2/n=\pi/4=0.785$, for which value $P=0.55$, and $1-P=0.45$ which is the probability that the resultant is greater than the average value.

Possibly of greater significance is the probability of a resultant greater than, say, one and one half times the average value. Here $r=1.5\bar{r}$, $r^2/n=1.77$, and $1-P=0.16$. Thus about 16% of the time the actual noise amplitude would be greater than 1.5 times the average noise amplitude.

APPLICATION TO SIGNAL-TO-NOISE RATIO

Assuming that several geophones on a single channel can be so placed that the signal, or reflection, arrives at all simultaneously, and that the noise arrives at random, it is possible to arrive at an expression for the expectancy of gain in signal-to-noise ratio. If ξ_s denote the signal amplitude, X_s the amplitude at each geophone, and n the number of geophones, then

$$\xi_s = n \cdot X_s. \tag{8}$$

The noise amplitude expectancy is a function of the noise arriving at each geophone and the number of geophones. If ξ_i is the noise amplitude to be expected, or the average value of the noise amplitude, and X_i the noise amplitude at each geophone, then, from (4),

$$\xi_i = X_i \cdot \frac{\sqrt{\pi}}{2} \cdot \sqrt{n}. \tag{9}$$

The signal-to-noise ratio with a single geophone per channel would be

$$R_1 = \frac{X_s}{X_i} \tag{10}$$

and the expected signal-to-noise ratio with n geophones per channel would be

$$R_n = \frac{\xi_s}{\xi_i}. \tag{11}$$

The gain in signal-to-noise ratio resulting from changing from 1 to n geophones per channel would have the expected value

$$\begin{aligned} \overline{G} &= \frac{R_n}{R_1} \\ &= \frac{2}{\sqrt{\pi}} \cdot \sqrt{n}. \end{aligned} \tag{12}$$

It should be remembered that the expected value of gain, $\overline{G}$, is subject to the same statistical variation as the average value of the resultant, $\bar{r}$. That is to say, the probability of an actual gain, G, at some instant, less than $\overline{G}$, is the same as the probability of a resultant, r, greater than $\bar{r}$. Approximately half the time, the gain, G, will be less than $\overline{G}$.

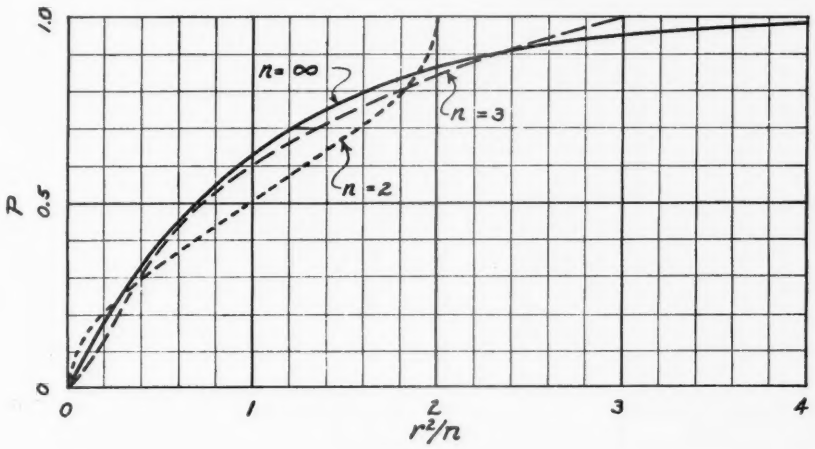

FIG. 3

SMALL VALUES OF n

Rayleigh's analysis (leading to the curve of Fig. 3) was based on the assumption that n is large. This immediately raises the question, "how large must n be so that the statistical analysis is at least approximately correct?"

The statistical distribution (similar to Fig. 3) has been worked out

for $n=2$ and $n=3$. Beyond this the problem becomes exceedingly difficult, involving finding the "volumes" of n dimensional spaces. Fortunately, however, a considerable convergence is indicated for the value of $n=3$. These two cases will now be taken up in detail.

DISTRIBUTION OF PROBABILITY OF RESULTANT OF TWO UNIT VECTORS

Assume 2 unit vectors at angles θ_1 and θ_2 with respect to an arbitrary reference line.

The resultant is

$$r = [(\cos\theta_1 + \cos\theta_2)^2 + (\sin\theta_1 + \sin\theta_2)^2]^{1/2}$$

$$= [2 + 2\cos(\theta_1 - \theta_2)]^{1/2}. \tag{13}$$

By definition, all values of θ_1 and θ_2 are equally probable. Then the probability that the resultant is less than r is

$$P = \frac{\iint_R d\theta_1 d\theta_2}{\iint_T d\theta_1 d\theta_2} \tag{14}$$

where R represents a region of values of θ_1 and θ_2 within which the resultant is less than r and T represents a total region. Equation (14) is one of the fundamental equations of probability.[5] Since values of θ_1 and θ_2 greater than 2π represent repetitions it is sufficient to take T as the limits of θ_1 and θ_2 from 0 to 2π or from $-\pi$ to $+\pi$ in which case the denominator of (14) becomes simply $4\pi^2$.

The numerator of (14) is evaluated for different values of r by rearranging (13):

$$2\cos(\theta_1 - \theta_2) = r^2 - 2$$

$$\theta_1 = \theta_2 + \cos^{-1}\left(\frac{r^2}{2} - 1\right). \tag{15}$$

The arc-cosine term has 2 values between 0 and 2π so the first integration (on θ_1) is performed between the limits

$$\theta_2 + \cos^{-1}\left(\frac{r^2}{2} - 1\right)$$

and

$$\theta_2 + 2\pi - \cos^{-1}\left(\frac{r^2}{2} - 1\right)$$

[5] Coolidge, loc. cit. page 10.

or simply

$$\theta_2 - \cos^{-1}\left(\frac{r^2}{2} - 1\right).$$

The double integration in the numerator is equivalent to finding the area represented by the shaded part of Fig. 4a. To simplify the limits of integration Fig. 4a may be rearranged in the form of Fig. 4b in

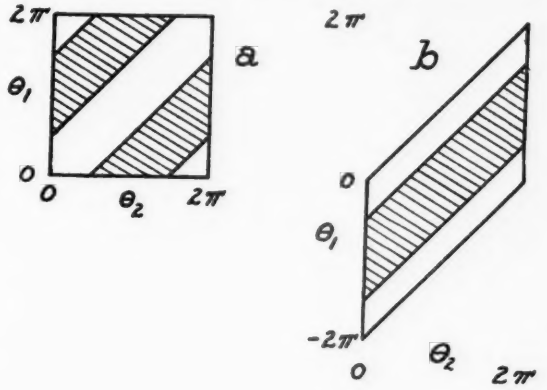

FIG. 4

which the areas are equivalent. Integration of θ_2 takes place between 0 and 2π so (14) becomes

$$P = \frac{\int_0^{2\pi}\int_{\theta_2-\cos^{-1}(r^2/2-1)}^{\theta_2+\cos^{-1}(r^2/2-1)} d\theta_1 d\theta_2}{4\pi^2} \tag{16}$$

$$= \frac{\int_0^{2\pi} 2\cos^{-1}\left(\frac{r^2}{2} - 1\right) d\theta_2}{4\pi^2}$$

$$= \frac{1}{\pi}\cos^{-1}\left(\frac{r^2}{2} - 1\right) \tag{17}$$

where P is the probability of obtaining a resultant less than r. Equation (17) is plotted in Fig. 3, curve $n=2$.

DISTRIBUTION OF PROBABILITY OF RESULTANT OF THREE UNIT VECTORS

In the case of 3 unit vectors the resultant is

$$r = [(\cos\theta_1 + \cos\theta_2 + \cos\theta_3)^2 + (\sin\theta_1 + \sin\theta_2 + \sin\theta_3)^2]^{1/2}$$

$$= [3 + 2\cos(\theta_1 - \theta_2) + 2\cos(\theta_1 - \theta_3) + 2\cos(\theta_2 - \theta_3)]^{1/2} \tag{18}$$

and the probability of a resultant less than r is

$$P = \frac{\iiint_R d\theta_1 d\theta_2 d\theta_3}{\iiint_T d\theta_1 d\theta_2 d\theta_3}. \tag{19}$$

The denominator of (19) is simply $8\pi^3$. Since the numerator is not amenable to the analytic methods used in (14), resort was had to graphical methods.

Equation (18), when studied geometrically, leads to a nest of cylinders; holding any one angle constant, as a parameter, gives a trace of the cylinders in a plane, and then permitting the parameter to vary generates a solid. This leads to the conclusion, which could have been arrived at intuitively, that one of the angles can be set equal to zero and the volume of a solid reduced to the area of a plane surface; this amounts to letting the arbitrary reference line follow one of the vectors. An analogy for the two vector cases is that in Fig. 4b, the length of a line instead of the area of a surface could have been used as a measure of the probability. This amounts to saying that (13) reduces to

$$r = (2 + 2\cos\theta)^{1/2} \tag{13b}$$

and (14) reduces to

$$P = \frac{\int_R d\theta}{2\pi} \tag{14b}$$

where the R region is simply a length of line. In like manner, (18) reduces to

$$r = [3 + 2\cos(\theta_1 - \theta_2) + 2\cos\theta_1 + 2\cos\theta_2]^{1/2} \tag{18b}$$

and (19) to

$$P = \frac{\iint_R d\theta_1 d\theta_2}{4\pi^2}. \tag{19b}$$

Holding r constant in (18b) gives rise to a closed curve, the area of which, divided by $4\pi^2$, is the probability that the resultant lies between o and r.

Equation (18b) is plotted in Fig. 5. Measurement of areas results in the probability curve for $n=3$ in Fig. 3.

ANALYSIS FOR LARGE VALUES OF n HOLDS FOR SMALL VALUES

Since as n increases, convergence must occur on the curve for large values of n, and since the curve for $n=3$ is a fair fit to the curve for $n=\infty$, it is to be inferred that for $n>3$ the curve for large values of n applies to a very close degree of approximation; that is, the statistical analysis for large values of n applies closely for all values of n greater than three.

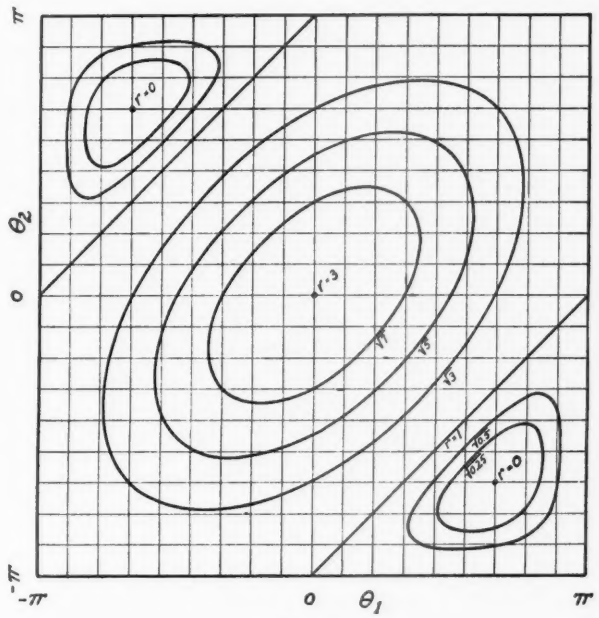

FIG. 5

PRACTICE

Equation (12) shows that the average gain in signal-to-noise ratio is approximately

$$\overline{G} = 1.13\sqrt{n}.$$

Evidently, to obtain an average gain of between 11 and 12 would require 100 geophones per channel, or 600 for a 6 channel recorder. This assumes that other factors, such as depth to which shots and geophones are buried, are not altered. Overlooking the obvious absurdity, consider the economics of placing a large number of geophones, or shots, below the irregularly attenuating and retarding topsoil, in order that all factors other than the number of units may be

the same. Twelve hundred feet of geophone holes is obviously out of the question.

On the other hand, suppose the cost be avoided by placing the geophones on the surface. Then the top-soil characteristics will frequently produce errors in reflection time and step-out as great as would be caused by ten times as many feet of ordinary weathering. The low-pass filter characteristics of a soft topsoil will also reduce or even eliminate the irregularities known as phase breaks by which a particular half cycle of a reflection is best identified for correlation purposes. Such loss in detail may more than offset the theoretical advantage.

Equation (12) may be regarded as establishing a maximum gain to be expected from the compounding. An assumption on which the analysis is based was that a configuration is possible by which the signal is recorded in phase and the noise out of phase. It should be realized that such a configuration may not be possible in all cases. For example, a reflection from a dipping bed, shot down-dip, may have a very low apparent surface velocity, and an array spread which is sufficiently great to cause the noise to be random may receive the signal with such phase lags that the signal amplitude is less than n times that for a single geophone. In such a case, the gain would be materially less than given by (12). Incidentally, records shot up-dip should generally be expected to display better gain in signal-to-noise ratio than records shot down-dip.

PREVIOUS REFLECTIONS AS A SOURCE OF NOISE

A type of noise so far not considered is that produced by several successive reflections occurring at such close intervals that they tend to mask each other. Suppose that a multiple shot or geophone system were so perfect that all noise of the irregular variety were completely eliminated. Whatever unwanted ground motion exists, of the nature of ground roll or refraction energy, is assumed to be reduced to a level below perceptibility. Only reflections would be recorded. If several reflections occur in quick succession, each reflection leaving behind it a decaying wave train, interference between reflections will occur which will tend to mask the information obtainable from each taken separately. That is, the decaying wave train of a reflection will constitute an interfering signal, or to all intents and purposes a noise, as far as the succeeding reflection is concerned. At least in the Gulf

Coast, where reflections appear on the records in great numbers, this would prove serious.

It is to be inferred, then, that what is regarded as a signal at one instant becomes at some later time a noise. Being part of a signal, it cannot be eliminated by a system which merely discriminates against random noise. Evidently, the multiple system cannot produce any advantage in the solution of a problem of this kind.

ACKNOWLEDGMENT

The writer is indebted to Mr. Ulrich of the Rice Institute, and to Drs. Blau, Slotnick and Rust of the Humble Company for suggestions and criticisms.

REVIEWS

Prospeccao Geophysica em São Paulo. By Irnack Carvalho do Amaral and Henrique Capper Alves de Sousa. Bulletin 10 of the Department of Mineral Production, Republic of Brazil, Rio de Janeiro, 1936. (In Portuguese.)

Part I deals with the magnetic instruments, methods and anomalies found in the areas of São Pedro-Xarqueada. Askania vertical and horizontal intensity magnetometers, and stationary instruments for photographic registration of diurnal and other changes, were used. Most of the anomalies were found to be related to diabase sills, lacoliths and dikes. Depth calculations by the methods of Tiberg, Haanel and Thalen did not correspond closely with depths found in wells, but were of the same order of magnitude. Depth to basement rocks could not be calculated from the magnetic results, and the authors suggest the use of seismic methods. Tables showing the magnetic properties of the rocks in the area are included.

Part II deals with the theory, instruments, field procedure and results of gravitational surveys in the São Pedro area. Small Bamberg balances were used. Porosity and density of rocks in the area are listed. Calculations for interpretation are made on the assumption that masses have an infinite extent in one direction. Maps showing gradients, curvature values, and isogams are presented for the São Pedro and adjacent areas.

Part III deals with a magnetic reconnaissance in the vicinity of Pitanga and Pau D'Alho, where anomalies are related to diabase sills, dikes, and lacoliths, and to structural elevation of rocks of relatively high permeability.

This publication of 102 pages contains little new in the way of theory. It will be of interest to those studying the geology of Brazil, and to those engaged in magnetic surveys involving intrusive and extrusive igneous rocks. Requests for the publication should be addressed to Bibliotheca do Departamento Nacional da Produccao Mineral, Avenida Pasteur 404, Praia Vernelha, Rio de Janeiro, Brasil.

JOHN H. WILSON

"Deformation of Rocks under High Confining Pressures. I. Experiments at Room Temperature." *Journal of Geology*, Vol. xliv, no. 5, pp. 541–577. By David T. Griggs. (1936).

This article describes experiments on the flow and rupture of rocks under compression, tension, and torsion, while at the same time subjected to a high confining pressure supplied through a liquid surrounding the specimen. The hydrostatic pressure of this liquid could be measured very accurately and could be maintained constant. In addition, a "differential" stress was applied to the specimen, and the deformation was measured directly. By using the high pressure technique of P. W. Bridgman the confining pressure was carried up to 13,000 atmospheres, equivalent to a depth in the earth's crust of 28 miles, and four times that available to F. W. Adams in his pioneering experiments (1901–1917).

The experiments of Adams had demonstrated beautifully that under high confining pressures rocks behave plastically. However, he was unable to measure his pressures accurately, to hold them constant, or to rupture the rocks under a confining pressure and examine the fracture after the experiment. All these defects have been remedied in the present apparatus.

The results of the present experiments contradict the ancient geological dogma that under a sufficient confining pressure a rock would flow indefinitely like a viscous fluid. Instead, it was found that whatever the confining pressure, up to the limits available, a differential pressure great enough to cause flow, if maintained long enough, would always eventually produce rupture. Nor did rapid loading under these circumstances produce rupture at a less stress. On the contrary, the specimen ruptured at less stress, but with greater deformation, under slow loading. This is really to be expected from the first finding.

The ultimate strength, and the amount of deformation preceding rupture, were vastly increased by a high confining pressure. But the elastic limit, and the behavior below that point, were little affected. Quartz remained brittle up to the highest pressures available.

The article is illustrated by many beautiful photographs of ruptured or deformed specimens, and by photomicrographs of the rocks before and after deformation.

It is to be hoped that the author's plan to continue these experiments at higher temperatures, and at still higher pressures, will soon be carried out.

M. MOTT-SMITH

"Shearing Phenomena at High Pressures of Possible Importance for Geology." *Journal of Geology*, Vol. XLIV, pp. 653–669, 1936. By P. W. Bridgman.

Contributions from Bridgman's high-pressure laboratory at Harvard are always of interest. In the present article he discusses flow, and the shearing stresses necessary to produce flow under high pressure for a wide variety of substances. Rigidity, or the resistance to shearing stress, is one of the fundamental constants of the seismologist, and Bridgman here deals with shearing stresses under conditions of great pressure comparable to those under which seismic data are obtained. The experiments were made at room temperatures, but at pressures as high as 50,000 kg./cm.,^{2} corresponding to those prevailing at depths of 166 km. Few rocks now available for examination at the surface of the earth are likely to have been buried so deep.

"The phenomena found proved to be both varied and unexpected." Great pressure produced a many-fold increase in shearing strength, 10-fold for most metals, 900-fold (an extreme case) from paraffine. Even under conditions of extreme distortion crystal structure persisted and at high pressures polymorphic transitions occurred in many cases, some with notable volume changes. Repeated deformation increased shearing strength in some cases, decreased it in others. "In the case of graphite "all the lubricating qualities associated with its initially easy internal slip are eventually lost, and at the end graphite becomes an abrasive, actually rubbing metal off the hardened steel surfaces." Plastic flow in forty per cent of the cases was accompanied by deformation, by internal rupture, and rehealing. Permanent, non-reversible changes in density occurred, and even chemical reactions. A number of substances showed important time effects.

"The existence of these various time effects suggests crucial questions with regard to behavior in geologic times. We are here, of course, in a field of almost pure speculation; but there are examples of such abrupt freezing of internal mobilities with decreasing temperature or increasing pressure that I think we should not be frightened by the bogy of even geologic time, but in our speculations freely use such concepts as shearing strength, admitting that the apparent strength in 10,000,000 years may be a little lower than for laboratory experiments, or the strength for 100,000,000 years a little lower than for 10,000,000, and also recognizing that the difference between geological and labora-

tory behavior may be a strong function of the material. We must at any rate recognize that the existence of these time effects means that no numerical datum or descriptive concept formulated in simple language is sharply applicable to geological phenomena." "Geology is rapidly approaching a point where an ultimate problem which has been staring it in the face can no longer be sidestepped, namely, to determine the actual specific physical and chemical behavior of those materials which do actually constitute the earth's crust."

The reader must keep in mind the fact that all these experiments were conducted on very thin films of material, only a few thousandths of an inch thick, that the pressure applied was unilateral, and not the conditions of cubic compression and enhanced temperatures that exist in the depths of the earth.

C. MAYNARD BOOS

"Magnetic Investigations in Southwest Alabama" by J. Brian Eby and E. G. Nicar; *Geological Survey of Alabama*, Bulletin No. 43, University, Ala. (June, 1936).

The report covers the results of field work with the Askania-Schmidt vertical magnetometer done by the junior author. The area worked includes all that part of Alabama lying within sixty-five miles of the Alabama-Mississippi boundary between the Gulf of Mexico and the northern portions of Choctaw, Clarke, and Monroe counties. A map of that area, on a scale of three miles to the inch, showing anomaly values in gammas and locations of stations occupied, and contoured on an interval of twenty gammas, accompanies the report.

A few clear concise paragraphs on the topography and surface geology of the area, are followed by a discussion of the principles involved in magnetometer work and their application to this particular area. By way of illustrating these principles, reference is made to igneous plugs in Arkansas and Mississippi, and some salt domes and anticlines, all previously mapped with a magnetometer by the junior author. It is stated that the magnetic relief of the igneous plugs was always positive, ranging from 600 to 2600 gammas, that of salt domes negative, and that of other anticlines either positive or negative, the magnitude of relief in the last two classes not exceeding 44 gammas and often requiring correction for the effect of deeply buried igneous masses. The effect of paramagnetic Tertiary sediments in producing the magnetic pictures of these last two types is discussed.

Six magnetic anomalies are pointed out as a result of the magnetic work in Alabama. Two of these are of the high-relief, positive type. It is stated that they can be interpreted as deep-seated plugs of igneous rock. While it is said that such plugs may be either erosional remnants on the surface of the basement or actual intrusions into the sediments, the authors neglect to point out that the plugs might also be basic intrusions or inclusions in an acid basement, which would not necessarily rise above the general level of the basement and thus might have no effect on the structure of the overlying sediments.

The other four anomalies explicitly referred to are marked by closures of twenty to forty gammas or by a mere spreading of the contours. Several other such anomalies shown on the map are not mentioned in the text. The authors point out that on the flanks of the major magnetic highs one could scarcely expect to find minor closures without correcting for the rapid lateral increase of magnetic intensity due to the major feature. Finally, they state that the general magnetic picture apparently is not

affected by two previously known geological structures: the Hatchetigbee anticline and the Jackson fault. Eight well logs from the region are appended.

Since the average density of magnetic stations read is only about one for each five square miles, the twenty gamma contour interval used appears excessively small to the reviewer. It has the effect of showing up small closures at intersections of the traverses run (on which stations were placed at one-mile intervals) and tends to give the impression of a featureless field within the polygons between the traverses, although doubtless several such small closures also exist within those polygons. The multiplicity of contours may somewhat obscure a rather interesting regional relationship shown by the map, namely, that the main streams of the region and their estuary in Mobile Bay follow in a general way the magnetic lows and distinctly avoid the major magnetic highs. This could be interpreted as meaning that the regional magnetic picture reflects in a rough way the configuration of the surface of the basement, and that the central drainage system of Alabama is geologically at least as old as early Cretaceous, occupying a geosyncline with, in general, a north-south axis.

JOSEPH L. ADLER

"Some Magnetometric and Gravimetric Surveys in the Transvaal." By O. Weiss, D. T. Simpson, and G. L. Paver, Department of Mines of the Union of South Africa, Geological Series Bull. No. 7, 1936, pp. 27, plates 8.

This short but interesting report covers test application of the magnetic and gravitational methods of prospecting to geologic problems in seven areas in South Africa:

1) Magnetic traverses across the Lower Witwatersrand System. Anomalies ranging between + and −10,000 gammas, close relation between anomalies and geology were observed.
2) Torsion balance survey across the Doornkop Fault; a text book example of gradient and differential curvature anomaly.
3) Torsion balance survey across the Witpoortje Fault and Lower Witwatersrand beds; an example of anomalies which are easy to interpret if you know the geology.
4) Successful determination of the hade of the Witpoortje Fault by a magnetic survey.
5) Torsion balance survey for locating nickel ore pipes. Brilliant indication is given of the pipes in spite of a regional gradient of more than 50E.
6) A Magnetic survey over Gossan, indicating the usability of magnetic methods in prospecting for nickel ore pipes.
7) Magnetic survey with negative results over chrome seams.

This is a valuable collection of cases of the application of these two methods.

D. C. BARTON

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Eugene McDermott, J. E. Jonsson, Barney Fisher
Sherwin Finch Kelly
Noel H. Stearn, G. H. Westby, T. A. Manhart
Helmut Klaus
Paul Weaver, John F. Weinzierl, Olaf F. Sundt
Karl Ewart Lewis
Barney Fisher, J. E. Jonsson, Eugene McDermott
Charles Lorimer Mackey
H. B. Peacock, Eugene McDermott, J. C. Karcher
Charles Culberson Mason
J. S. Watt, W. D. Mounce, W. W. Rust, Jr.
Reed L. McCartney
E. E. Rosaire, F. M. Kannenstine, E. Stiles
Downs McCloskey
B. B. Weatherby, Wm. T. Born, L. Y. Faust
John Rufus McMahan, Jr.
W. W. Rust, Jr., W. D. Mounce, J. S. Watt
Henry Silliman McQueen
J. S. Watt, R. R. Thompson, M. A. Arthur
James C. Menefee
J. S. Watt, W. W. Rust, Jr., W. D. Mounce
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W. E. McDermott, Eugene McDermott, J. C. Karcher
James A. Moore
W. Harlan Taylor, T. L. Allen, O. S. Petty
Elay Kenton Myers
R. D. Wyckoff, Paul Weaver, E. A. Eckhardt
Leo James Peters
T. A. Manhart, Paul Weaver, E. A. Eckhardt
Pedro Rey
E. E. Rosaire, Derry H. Gardner, Louis Statham
Terence Charles Richards
J. H. Jones, John H. Wilson, A. L. Smith
John Frederick Riddle
J. S. Watt, R. R. Thompson, Louis Statham

Frederick Ernst Romberg
L. G. Howell, W. W. Rust, Jr., J. S. Watt
William Schriever
J. S. Watt, R. R. Thompson, M. A. Arthur
Marcel Schlumberger
E. G. Leonarden, E. E. Rosaire, F. M. Kannenstine
Ellis William Shuler, Jr.
Eugene McDermott, J. C. Karcher, W. E. McDermott
Frank Benjamin Smith
O. S. Petty, T. L. Allen, W. Harlan Taylor
John Barron Souther
W. W. Rust, Jr., J. S. Watt, Louis Statham
Melvin Carhart Terry
J. S. Watt, W. W. Rust, Jr., W. B. Lewis
William Edmund Wallis
G. F. Kaufman, Dave P. Carlton, J. E. LaRue
Waldo William Waring
J. S. Watt, Leland L. Antes, A. Frosch
Loren Elvin Whitehead
L. F. Athy, L. A. Scholl, Jr., E. V. McCollum
Joe Tracy Williams
J. S. Watt, Leland L. Antes, A. Frosch
Floyd Willis, Jr.
M. C. Born, W. T. Born, J. E. Owen
Robert Ellerston Woods
Roy F. Bennett, A. L. Smith, John H. Wilson
Hughes Mead Zenor
J. S. Watt, W. W. Rust, Jr., W. D. Mounce

FOR ASSOCIATE MEMBERSHIP

Willis Oliver Bazhaw
T. L. Allen, O. S. Petty, W. H. Taylor
Richard Everett Blight
E. P. Evans, O. C. Lester, L. Y. Faust
Richard Bunting Coit
Eugene McDermott, J. C. Karcher, J. E. Jonsson
Jack E. Fore
E. E. Rosaire, J. W. Flude, T. I. Harkins
Lawrence Frank Guseman
C. C. Zimmerman, L. A. Scholl, Jr., C. T. McAllister
Audio Gray Harvey
Eugene McDermott, J. E. Jonsson, Barney Fisher
Hubert Slocum Harvey
Derry H. Gardner, L. Atmar M. Barnette, J. S. Watt
Jesse Duff Hatch
Derry H. Gardner, L. Atmar M. Barnette, J. S. Watt
Homer Lafayette Heggy
W. R. Ransone, Barney Fisher, J. E. Jonsson

Walter Johnson
Robert Saibara, E. E. Rosaire, J. W. Flude
Cecil Klemme Jordan
L. F. Athy, L. A. Scholl, Jr., E. W. McCollum
Ernest August Kiesler
Barney Fisher, Eugene McDermott, J. C. Karcher
Kenneth Morrison Lawrence
O. C. Lester, Jr., John L. Ferguson, L. Y. Faust
Manuel Calva Lecuona
J. M. Adler, H. W. Falkenhagen, J. M. Golden
William Isaac Mayfield
T. L. Allen, O. S. Petty, W. H. Taylor
Marion Jefferson Moore
T. L. Allen, O. S. Petty, W. H. Taylor
Pedro Eugenio Narvarte
T. L. Allen, O. S. Petty, W. H. Taylor
Robert George Nisle
H. B. Peacock, Eugene McDermott, J. C. Karcher
James Angelo Smith
W. M. Rust, Jr., W. B. Lewis, J. S. Watt
Elden Edward Stroshine
W. Harlan Taylor, T. L. Allen, O. S. Petty
Brunislaus Stephen Ulak
John W. Flude, E. Stiles, C. Maynard Boos
Daniel Rey Vercesi
W. G. Green, T. A. Manhart, F. B. Leedy
Joe LaFayette Whittenberg
J. S. Watt, Leland L. Antes, A. Frosch
Robert Edward Wilder, Jr.
E. E. Rosaire, T. I. Harkins, J. L. Lake, Jr.

EXECUTIVE ORDER NO. 21

CHANGE OF STATUS

By authority of Article III-B-6 of the constitution, and in accordance with their Executive Order No. 21, the Executive Committee has advanced the following from associate active to membership:

B. O. Winkler
D. F. Broussard

EXECUTIVE ORDER NO. 18

FOREIGN MEMBERSHIP FEES AND SUBSCRIPTIONS

The Executive Committee has authorized the addition of fifty cents per year to the membership fee for foreign members and fifty cents per year to the subscription price to foreign addresses to take care of additional postage, loss in exchange, and handling charges incident to such memberships and subscriptions. Individual numbers will be billed to foreign addresses at regular prices plus twenty cents per number.